Module 4
Tracking Cells using the Particle Filter

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Virginia Image and Video Analysis

Developing Leaders of Innovation

"I find that the harder I work, the more luck I seem to have."

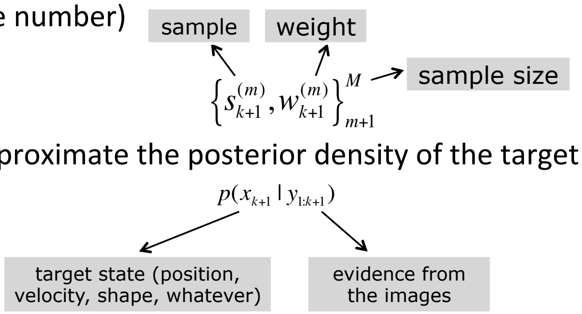
- Thomas Jefferson (1743-1826)

Goal

- Instead of one "best guess," pursue a multitude of "guesses" about the target state
- What's presented here
  - A simple particle filter implementation useful for cell tracking
- Not presented here
  - The "full breadth" of particle filter methods – this example is a Monte Carlo approach to tracking…

Particle Filter Tracking

- Use a sample set (these are the particles!) (*k* is the current frame number)



sample

weight

sample size

$$\left\{ s_{k+1}^{(m)}, w_{k+1}^{(m)} \right\}_{m+1}^{M}$$

to approximate the posterior density of the target state

$$p(x_{k+1} \mid y_{1:k+1})$$

target state (position, velocity, shape, whatever)

evidence from the images

- For each new frame in a video sequence, we:
  - Generate samples of the target state (e.g., position) using a motion model
  - Compute sample likelihood by the observation model

PF Tracker (an example for biological imaging)

- Position prediction
  - horizontal movement does not change dramatically (assumption: cell is moving horizontally)
  - vertical movement is negligible

predicted position

$$\bar{x}_{k+1} = \hat{x}_k + \alpha(\hat{x}_k - \hat{x}_{k-1}) + (1-\alpha)(\hat{x}_{k-1} - \hat{x}_{k-2})$$

$$\bar{y}_{k+1} = \hat{y}_k$$

estimated position

- Sample generation
  - Generate samples randomly with a 2-D Gaussian distribution (for example)

$$x_{k+1}^{(m)} = \bar{x}_{k+1} + R\cos\beta \qquad R \sim N(0, \sigma_R^2)$$

$$y_{k+1}^{(m)} = \bar{y}_{k+1} + R\sin\beta \qquad \beta \sim U(0, 2\pi)$$

PF Tracker

- Radial edge detection
  - construct N line segments extending radially from each sample
  - specify K+1 points on each line segment

$$x_{i,j} = x_{k+1}^{(m)} + [r_1 + (r_2 - r_1)\, j/K]\cos(2\pi i/N) \qquad i = 0, 1, \ldots, N-1$$

$$y_{i,j} = y_{k+1}^{(m)} + [r_1 + (r_2 - r_1)\, j/K]\sin(2\pi i/N) \qquad j = 0, 1, \ldots, K$$

  - apply edge detection operator on each line segment

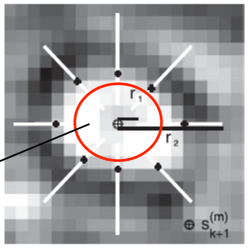
$$e_{i,j} = I_{i,j-1} + 2I_{i,j} - 2I_{i,j+1} - I_{i,j+2}$$

  - obtain local image intensity measurement

$$r_{\theta,k+1}^{(m)} \qquad \sigma_{I,k+1}^{(m)}$$

detected radius in each radial direction

intensity standard deviation of detected target interior



PF Tracker

- Sample weighting
  - assume target center position in the first frame is known
  - for the first frame, apply radial edge detection at the center position to obtain image intensity measurement as reference

standard deviation of radius measurement from 1st frame
↗
average radius from 1st frame ← $\bar{r}$ σ_r σ_I → intensity standard deviation of detected target interior from 1st frame

  - for each sample in a new frame, define difference measures

$$d_{1,k+1}^{(m)} = \left| \sum_{\theta} \left(r_{\theta,k+1}^{(m)} - \bar{r} \right)^2 - N\sigma_r^2 \right| \qquad d_{2,k+1}^{(m)} = \left| \sigma_{I,k+1}^{(m)} - \sigma_I \right|$$

  - assign a normalized weight to each sample

$$z_{k+1}^{(m)} = e^{-d_{1,k+1}^{(m)2} / 2\sigma_1^2} e^{-d_{2,k+1}^{(m)2} / 2\sigma_2^2} \qquad w_{k+1}^{(m)} = \frac{z_{k+1}^{(m)}}{\sum_{i=1}^{M} z_{k+1}^{(i)}}$$

A Monte Carlo Tracker

- Target position estimation
  - weighted sum of the sample set is the estimated target position

$$\left(\hat{x}_{k+1}, \hat{y}_{k+1} \right) = \sum_{m=1}^{M} w_{k+1}^{(m)} s_{k+1}^{(m)} \qquad s_{k+1}^{(m)} = \left(x_{k+1}^{(m)}, y_{k+1}^{(m)} \right)$$

END

- We showed one possibility of using the particle filter for tracking cells

*example in PFTracker