Image Segmentation by Curve Evolution with Clustering

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Abstract

A generalized approach to image segmentation is presented in this paper. The approach consists of two successive stages. First, a fuzzy c-means clustering algorithm is used to separate the image pixels into N classes. Second, a curve evolution based on partial differential equations is utilized to subdivide the image into an arbitrary number of closed regions. Our segmentation method uses level-set theory to evolve geometric snakes that delineate the image regions. The partial differential equations that govern the snake evolution are a steepest descent solution to an energy functional that penalizes both region inhomogeneity and increased segmentation boundary length.

1 Introduction

Fuzzy *c*-means clustering is an established classification technique [1]. It is well known that such a clustering process fails to delineate meaningful segments from an image in the presence of noise. Moreover the clustering process itself introduces noise in the image classification [2]. In this paper we have introduced a curve evolution method to detect meaningful segments from the classified image. The curve evolution eliminates noise that might arise from the imaging process and/or the clustering process. For the curve evolution technique we rely on the level set curve evolution method introduced by Osher and Sethian [3]. There is an equally important aspect introduced in the paper: this work can be viewed as image segmentation through evolving geometric snakes / active contours. It is well known that all kinds of active contours whether parametric or geometric [5] suffer from the problem of initialization. Remarkable work that provides a solution for the initialization process includes that of Zhu and Yuille [6] and more recently Yezzi *et al.* [7]. In this paper we offer a solution to this problem through classifying the pixels by fuzzy *c*-means algorithm.

2 Background

We combine two well-established techniques, *viz.* fuzzy *c*-means classification and curve evolution by level set method to segment images. The procedure of image segmentation consists of two successive steps in the proposed method. In the first step the image is classified into N classes. In the next step we evolve curves on the classified image to capture each of N regions. We have used fuzzy *c*-means clustering technique for the first step. This clustering algorithm is a standard version [2]. The second step introduces curve evolution via the level set method.

2.1 Fuzzy *c*-means classification

Let us assume that we want to classify the pixels of an image $I(x,y)$ into N classes. Each pixel at position (x,y) has the fuzzy membership value for the i^{th} class, $u_i(x,y)$. The clustering technique is based on minimizing an objective functional that accounts for the distance between cluster centers and the data (here it is the pixel value) within various clusters. The objective functional in this case is given by,

$$J(U,\mathrm{M}) = \sum_{x,y}\sum_{i=1}^{N}(u_i(x,y))^m |d_i(x,y)|^2 \quad (1)$$

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where U is the fuzzy c-means class partition, M is the set of cluster centers, and $m>1$, is the fuzzy exponent. For a pixel value $I(x,y)$, the measure,

$$|d_i(x,y)| = |I(x,y) - \mu_i| \quad (2)$$

is the distance between the pixel value and the cluster center, μ_i for the i^{th} class.

As seen from the equation (1) this distance is weighted in the objective functional by the fuzzy membership value for the corresponding pixel. The objective functional is iteratively minimized, subject to the following conditions:

$$\sum_{i=1}^{N} u_i(x,y) = 1, \text{and}\, u_i(x,y) > 0. \quad (3)$$

At the start of the iteration scheme, fuzzy membership values are assigned random values and normalized for each pixel to unity. Next the iteration scheme proceeds as the following two quantities are computed one after another in each step:

$$\mu_i = \frac{\sum_{x,y} (u_i(x,y))^m I(x,y)}{\sum_{x,y} (u_i(x,y))^m}. \quad (4)$$

and

$$u_i(x,y) = 1/[\sum_{x,y}\sum_{l=1}^{N}\left(\frac{d_i(x,y)}{d_l(x,y)}\right)^{m/(m-1)}], \quad (5)$$

Convergence is defined by the insignificant change between the two successive values of the objective functional. The output of the scheme is the partition U of the pixels into N classes.

2.2 Level set curve evolution

Sethian *et al.* [3,8] have conceived an evolving 2-D curve as the intersection-set of a plane (the plane on which the curve resides) and a 3-D conical surface. As time passes on, the cone evolves outward/inward changing its shape. As a result the curve *i.e.*, the intersection set evolves with time t. If $\Phi(x,y,t)$ is the evolving 3-D cone, then, the intersection set of points (x,y) such that $\Phi(x,y,t) = 0$ is typically termed as *zero level set* at any instant t. So, by tracking the evolving surface, one can easily compute the evolving curve or the zero level set at any moment. This technique has a great advantage: two such evolving curves can be easily merged and equivalently an evolving curve can be easily split into two or more curves. In other words, this model allows changes in the topology change of curves [8]. The evolution equation of the zero-level set is given by

$$\frac{\partial \Phi}{\partial t} + F|\nabla\Phi| = 0, \quad (6)$$

where $F(x,y)$ is the speed of the curve evolution in the direction to the outward normal of the curve at (x,y). Equation (6) is the level set curve evolution equation given by Sethian [8]. We derive the speed function F in the next section by minimizing an energy functional that penalizes the length of the segments (edge lengths) and penalizes the region inhomogeniety after image classification.

3 Energy based curve evolution

Let us say that we are given N closed regions (curves) $\mathbf{C}_1$, $\mathbf{C}_2$, ...$\mathbf{C}_N$ on the image plane. We define a mapping from $\mathbf{C}_i$ to $\Re^N$ in the following way:

$$g(\mathbf{C}_i) = (card(\mathbf{S}_{i,1}),...,card(\mathbf{S}_{i,N}))/card(\mathbf{C}_i),$$
$$\forall i = 1..N, \text{ where,}$$
$$\mathbf{S}_{i,j} = \left\{\begin{matrix}(x,y):(x,y)\in \mathbf{C}_i \text{ and } (x,y) \text{ is} \\ \text{classified in } j^{th} \text{ class}\end{matrix}\right\},$$
$$\forall j = 1..N.$$

So ideally, $g(\mathbf{C}_i) = \mathbf{e}_i$, where, $\mathbf{e}_i$ is the i^{th} Euclidean basis vector in $\Re^N$. One then tries to maximize i^{th} component of $g(\mathbf{C}_i)$ and minimize j^{th} component of $g(\mathbf{C}_i)$, $\forall j \neq i$. This gives the motivation behind the following set of N energy functionals which is to be minimized:

$$E(\mathbf{C}_1(p(s),q(s))) = \iint_{\mathbf{C}_1} f_1(x,y)dxdy + \alpha \int_{\partial\mathbf{C}_1} \sqrt{p_s^2+q_s^2}\,ds,$$
$$\vdots \quad (7)$$
$$E(\mathbf{C}_N(p(s),q(s))) = \iint_{\mathbf{C}_N} f_N(x,y)dxdy + \alpha \int_{\partial\mathbf{C}_N} \sqrt{p_s^2+q_s^2}\,ds,$$

where, $s \in [0,1]$ is the parameter for active contour, $\alpha > 0$ is a parameter that penalizes the length of the corresponding curve and $f_i(x,y)$ is defined as follows:

$$f_i(x,y) = -1, \text{ if } (x,y) \in i^{th} \text{ class},$$
$$= 1, \text{ otherwise.}$$

From (7) we get a set of N decoupled energy descent equations to evolve the curves. It can be shown that the evolution speed F (as in equation (6)) for the i^{th} curve becomes

$$-f_i(x,y) - \alpha\kappa_i, \text{ where, } \kappa_i = \mathrm{div}\left(\frac{\nabla\Phi_i}{|\nabla\Phi_i|}\right).$$

4 Unambiguous evolution with a particular initialization

In order to solve level set evolution equation, one has to initialize the values of the evolving cone Φ, as it is an initial value problem [8]. Here the active contour is initialized with a simple strategy by exploiting the fuzzy c-means classification. For example, to initialize for class i,

$$\Phi_i(x,y,0) = -D, \text{ when, } (x,y) \in \text{class } i$$
$$= D, \text{ otherwise.}$$

Here, D is the maximum value for Φ_i in the narrow-band evolution technique [5]. In other words, we take the boundaries of classification as the initial (t=0) active contour.

The ambiguity will occur in this independent framework only when after the energy minimization (curve evolution) any two curves assigned to two different classes will have some overlapping region. The initialization strategy ensures that there will be no ambiguity in this case. The initialization is done in such a way that no two curves belonging to two different classes (say, 1 and 2) overlap, at most they touch other at some common boundary. Let (x,y) be any point on this common boundary. This means the speed of (x,y) on the class 1 contour is $F_1(x,y) = -f_1(x,y) - \alpha\kappa_1(x,y)$ and speed of the same point on the class 2 contour is $F_2(x,y) = -f_2(x,y) - \alpha\kappa_2(x,y)$. Since, $\kappa_1(x,y) = -\kappa_2(x,y)$, we have two cases, (1) $f_1(x,y) = -f_2(x,y)$, in this case the contour move in the same direction with the same speed. (2) $f_1(x,y) = f_2(x,y) = 1$, in this case the curves move away from each other as $1 >> |\alpha\kappa_1(x,y)|$ and $1 >> |\alpha\kappa_2(x,y)|$. Hence, the curves can never cross each other. Since active contours belonging to different classes are essentially de-coupled this way, there exists a clear possibility of parallel evolution of the contours.

5 Results and discussion

Here we give some of the results of the active contour evolution. Figure 1(a) shows a coin image. Figure 1(b) shows the classified (2 classes) coin image and Figure 1(c) shows the proposed segmentation result. Figures 2(a), (b), (d) show a similar sequence after salt & pepper noise has been applied to the coin image. For comparison, we have given in Figure 2(c) the result of applying a 3x3 median filter to the classified noisy coin image. We have also applied the technique to segment a satellite image shown in Figures 3. Figures 3(a) and (b) are the original and the classified version (3 classes). Figures 3(c) and (d) show the result of the proposed curve-evolution at two different α values. This illustrates the scaling property of the evolution method. Figure 3(e) is the result of applying Perona-Malik anisotropic diffusion to the membership valued (multi-valued) image and then classifying the result of diffusion according to the highest membership value. The figure reveals that it is over smoothed. Figure 3(f) shows another post processing attempt with Gausian convolution. Here the noise is not properly removed.

Acknowledgements: We thank A. Neuenschwander and M. Crawford of the University of Texas for the LIDAR imagery.

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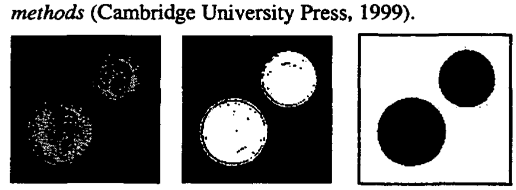

Figure 1 (a) Coin image. (b) Classified image (2 classes). (c) Curve evolution with clustering result at α=0.5.

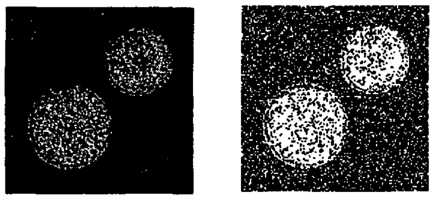

Figure 2 (a) Coin image with salt & pepper noise. (b) Classification based on 2 classes.

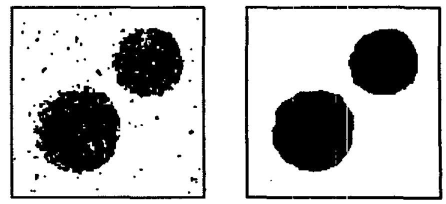

Figure 2 (c) Median filter on Figure 2(b). (d) Curve evolution with clustering result at $\alpha = 1.5$.

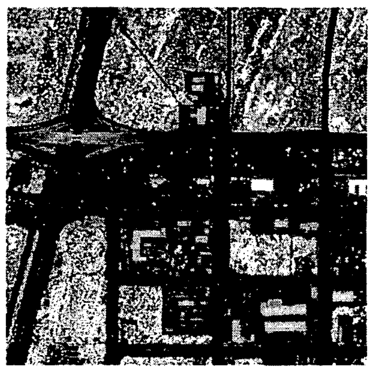

Figure 3 (a) A LIDAR image.

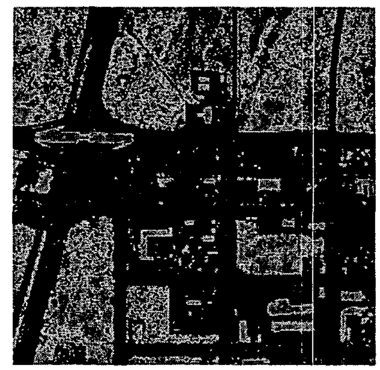

Figure 3 (b) Classified LIDAR (3 classes).

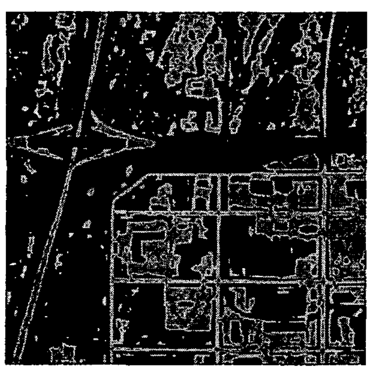

Figure 3 (c) Curve evolution with clustering based on 3 classes, at $\alpha = 0.2$.

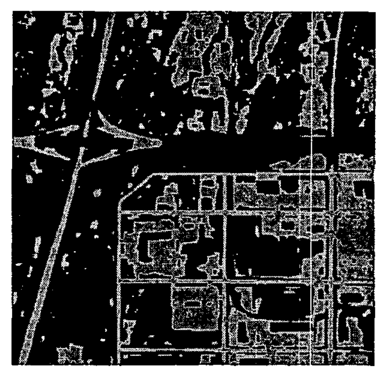

Figure 3 (d) Curve evolution with clustering based on 3 classes, at $\alpha = 0.5$.

Figure 3 (e) Perona-Malik anisotropic diffusion.

Figure 3 (f) Gaussian convolution