SELF-DUAL INCLUSION FILTERS FOR GRAYSCALE IMAGERY

Nilanjan Ray and Scott T. Acton
{nray, acton}@virginia.edu
Department of Electrical and Computer Engineering,
University of Virginia, Charlottesville, VA 22904

Abstract. Using the structure of an adjacency-tree for binary-valued images, we define inclusion filters, a class of connected operators. Inclusion filters modify the image by filling or retaining the holes of the connected components of foreground and those of the background of a binary image depending on application-specific criteria, which are increasing in the set theoretic sense. We demonstrate a straightforward method to achieve self-duality (gray level inversion invariance) of inclusion filters on the discrete Cartesian domain by considering only 8-adjacency. Inclusion filters are extended to the grayscale images by the threshold decomposition principle. As an application, inclusion filters are shown to improve the performance of snake-based tracking of leukocytes observed in intravital microscopic video imagery. In this set of experiments the mean position error is reduced by a factor of 2.5 using the inclusion filter.

I. INTRODUCTION

An important class of nonlinear filters is connected operators [1]. An operator is called connected if it deletes or retains (but does not distort) the connected components of a binary image or the connected components of the complement of a binary image [1,2]. The basic approach of such filtering methods is to decompose a grayscale image into level sets (binary images obtained by thresholding the grayscale image), filter the level sets, and then re-combine the filtered level sets [1,2]. Examples include flat zone filters, area open, area close, and grain filters [3]. The importance of such filters is manifested in many applications, such as segmentation [4], classification [5], and motion estimation [2].

The topographic map or the level line (boundary of a connected component within a level set) based representation of a grayscale image on a continuous domain essentially describes the same kind of contrast invariant connected operators that fills the holes of connected sets or retains them [6]. The key concept here is that a closed level line forms a hole and that level lines are nested and non-intersecting. Due to the existence of Jordan curve theorem for discrete domain imagery, the concept of hole filling can be directly adapted to the discrete Cartesian domain [7]. But, such direct adaptation of a level line based approach in the discrete Cartesian domain results in non-self-dual filters. (An operator t is self-dual if for any binary image, L: $\overline{t(L)} = t(\overline{L})$.) As an example, Figure 1(a) shows a checkerboard. Let us assume 8-adjacency for foreground (black) and 4-adjacency for background (white). If we want to fill all holes with area less than 2, then we obtain Figure 1(b). In Figure 1(c) we show the complement of Figure 1(a). Applying the same filter with the same adjacencies to Figure 1(c) results in no change. Now the complement of Figure 1(c) is Figure 1(a), which is not the same as Figure 1(b). So the grain filters [7] implemented on the discrete Cartesian domain are not self-dual.

Instead of relying on the level line based description, we show in this paper that the adjacency tree [3,8] approach to define such filters results in a perfectly self-dual filter, provided we consider *only 8-adjacency* for both foreground and background of an image level set. Figure 2(a) shows a binary image, where we label the connected components of foreground and those of the background, respectively in black and white. The adjacency forest (collection of adjacency trees) for Figure 2(a) is shown in Figure 2(b). The white and the black nodes of the adjacency tree respectively denote the connected components of background and those of foreground. The root nodes in the forest designate the connected sets touching the image boundary. The existence of the adjacency tree can be shown when any one of 4-8, 8-4, or 8-8 adjacency is considered [3,8,9]. By *n-m* adjacency we denote *n*-adjacency for the foreground and *m*-adjacency for the background on the discrete Cartesian domain. Now, to illustrate the concept of holes consider the connected set 1_W in Figure 2(a). The two holes of 1_W are $(3_B \cup 3_W \cup 4_W)$ and $(4_B \cup 5_W \cup 6_W \cup 7_W)$. Similarly the only hole of the connected set, 2_W, is 2_B. With the help of the adjacency tree we can also demonstrate the filling of a hole. Consider filling the two holes, 5_W and 7_W. The binary image and the corresponding adjacency forest after filling these holes are shown respectively in Figures 2(c) and 2(d). This leads to the proposed filter we refer to as inclusion filter: retain or fill the holes based on some criterion, which is an increasing function, mapping sets of points to the elements of the set {0,1}, where 0 means 'fill' and 1 means 'retain.' The inclusion filter paradigm may be viewed as a class of perfectly self-dual grain filters. An example of increasing criterion defining an inclusion filter is as follows: retain a hole if it contains any point belonging to the set 5_W or 7_W;

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otherwise, delete the hole. Filtering Figure 2(a) with this criterion, we obtain the result shown in Figure 2(c).

II. SELF-DUAL INCLUSION FILTERS

The existence of adjacency forest for any binary image establishes a very important fact: if a foreground (background) connected set is finite, then a unique background (foreground) connected set surrounds it [8].

We characterize the *holes* of connected sets by means of the adjacency forest as follows. Let S be a node in the adjacency forest. We denote by $F(S)$ the sub-tree rooted at S in the adjacency forest, *i.e.*, $F(S)$ denotes the set of points in the sub-tree rooted at S. Thus if $C_1, C_2, \dots, C_n$ are the children nodes of S, then we say S has n holes, and they are $F(C_1), F(C_2), \dots, F(C_n)$. Any unbroken path (a set of ordered adjacent points) from a point belonging to a hole, $F(C_i)$ of S, to any point belonging to the image boundary, must intersect S. Conversely, if all unbroken paths from a point to the image boundary always intersect S, then the point must belong either to S or to some hole, $F(C_i)$ of S. Filling a hole, $F(C_i)$ of S, means to change the value of the points belonging to $F(C_i)$ to the same value as the points of S. In other words, filling the hole $F(C_i)$ means to merge the sub-tree rooted at C_i with S. Thus, filling all the holes of S will result in the set, $F(S) = S \cup F(C_1) \cup \cdots \cup F(C_n)$. Here, any root node in the adjacency forest is called an *infinite connected set*.

The adjacency forest naturally characterizes the *inclusion sequence* defined as follows. Let L be a binary-valued image and $\mathbf{x}$ be a point in the image domain. Then,

(i) if $L(\mathbf{x}) = 1$, then there exists a unique alternating sequence of foreground and background connected sets – C_1, H_2, $C_3, \dots$, such that $\mathbf{x} \in C_1$ and H_2 is the parent of C_1, C_3 is the parent of H_2, and so on.

(ii) if $L(\mathbf{x}) = 0$, then there exists a unique alternating sequence of background and foreground connected sets – H_1, C_2, H_3, …, such that $\mathbf{x} \in H_1$ and C_2 is the parent of H_1, H_3 is the parent of C_2, and so on. Such a sequence will henceforth be called an inclusion sequence (IS). Any IS always terminates with an infinite connected set.

We first define an increasing criterion and then define inclusion filter with the use of inclusion sequences.

Increasing Criterion. Let Ω be the image domain, then a function, T, mapping the power set of Ω to the set $\{0,1\}$, is increasing if $0 \le T(S_1) \le T(S_2) \le 1$, for any $S_1 \subset \Omega, S_2 \subset \Omega$ with $S_1 \subset S_2$. Additionally we stipulate that $T(S) = 1$ for any infinite connected set S.

Inclusion Filter. Let $C_1, H_2, C_3 \dots$ or $H_1, C_2, H_3 \dots$ be an IS for $\mathbf{x}$ in the binary image L, where C's and H's respectively denote foreground and background connected sets. Now, $F(C_n)$ is a hole of H_{n+1}, $F(H_{n+1})$ is a hole of C_{n+2}, and so on. So, we have: $\dots \subset F(C_{n-1}) \subset F(H_n) \subset F(C_{n+1}) \subset \dots$ from the adjacency tree. For an increasing criterion T we have: $0 \le \dots \le T(F(C_{n-1})) \le T(F(H_n)) \le T(F(C_{n+1})) \le \dots \le 1$. Since the last term in this sequence is unity-valued, only the four mutually exclusive and exhaustive possibilities exist:

a) $T(F(C_1)) = T(F(H_2)) = T(F(C_3)) = T(F(H_4)) = \dots = 1$,

b) $\exists n$ such that $\dots = T(F(C_{n-1})) = T(F(H_n)) = 0$ and $T(F(C_{n+1})) = T(F(H_{n+2})) = \dots = 1$,

c) $T(F(H_1)) = T(F(C_2)) = T(F(H_3)) = T(F(C_4)) = \dots = 1$,

d) $\exists n$ such that $\dots = T(F(H_{n-1})) = T(F(C_n)) = 0$ and $T(F(H_{n+1})) = T(F(C_{n+2})) = \dots = 1$.

We define, $L_f(\mathbf{x})$, the inclusion filtered $L(\mathbf{x})$, as follows:

$$L_f(\mathbf{x}) = \begin{cases} 1, \text{ if (a) or (b) is true,} \\ 0, \text{ if (c) or (d) is true.} \end{cases}$$

In short we say $L_f(\mathbf{x}) = \psi_T(L(\mathbf{x}))$. It is easy to verify that inclusion filter is a formal description of filling the holes of connected sets of a binary image based on any increasing criterion. To prove the idempotency of the inclusion filter we first state and prove the following Lemma.

Lemma I. Let S_1 and S_2 be two connected sets such that $S_1 \supset S_2$, then $F(S_1) \supset F(S_2)$.

Proof. Let $\mathbf{p} \in F(S_2)$ and examine two cases. In the first case, $\mathbf{p} \in S_2$. Then $\mathbf{p} \in S_1$. Therefore $\mathbf{p} \in F(S_1)$. In the second case, $\mathbf{p} \notin S_2$, but, $\mathbf{p} \in H_i^2$, a hole of S_2. Then, any unbroken path, π from $\mathbf{p}$ to the image border meets S_2, *i.e.*, $\pi \cap S_2 \ne \phi$. Then, $\pi \cap S_1 \ne \phi$. So $\mathbf{p}$ belongs to a hole of S_1, *i.e.*, $\mathbf{p} \in F(S_1)$. Therefore $F(S_1) \supset F(S_2)$. <>

Proposition I. Inclusion filters are idempotent.

Proof. Let $L_f(\mathbf{x}) = \psi_T(L(\mathbf{x}))$. We first consider case (a). After filtering, the IS of $\mathbf{x}$ in L_f becomes $C_1', H_2', C_3', \dots$, with $C_1' \supset C_1, H_2' \supset H_2, C_3' \supset C_3, \dots$ Therefore, by Lemma I: $F(C_1') \supset F(C_1), F(H_2') \supset F(H_2), F(C_3') \supset F(C_3), \dots$ Thus, $T(F(C_1')) = T(F(H_2')) = T(F(C_3')) = \dots = 1$, so $L_f(\mathbf{x}) = \psi_T(L_f(\mathbf{x}))$, implying that the filter is idempotent in this case. In case (b), the hole $F(H_n)$ of C_{n+1} is filled. So after the inclusion filtering the IS for $\mathbf{x}$ becomes $C_1', H_2', C_3', \dots$, with $C_1' \supset C_{n+1}, H_2' \supset H_{n+2}, C_3' \supset C_{n+3}$, and hence, the same result follows. Cases (c) and (d) are similar. <>

Proposition II. Inclusion filters are self-dual.

Proof. Under the intensity reversal of a binary image, the nodes in the 8-8 adjacency forest change only their color (black to white and *vice versa*), but the entire forest remains unchanged in terms of its structure. This implies that if the IS of $\mathbf{x}$ in a binary image L is $C_1, H_2, \dots, (H_1, C_2, \dots)$ then the IS of $\mathbf{x}$ for the complement $\overline{L}$ of L is $H_1, C_2, \dots, (C_1, H_2, \dots)$, where C's and H's denote the foreground and the background connected sets. Therefore, from the definition of the inclusion filter: $\psi_T(L(\mathbf{x})) = \overline{\psi_T(\overline{L}(\mathbf{x}))}$. <>

Inclusion filtering can be extended to grayscale images by means of a threshold decomposition technique [1,2], where the input grayscale image is first decomposed into binary images (level sets) with increasing thresholds, then each of these binary images is filtered with inclusion filter, and finally the output binary images are stacked (added) to obtain the output grayscale image. In order for this technique to be meaningful, *gray level causality* must be maintained.

Gray level causality. Let L^{λ_1} and L^{λ_2} be two level sets of a grayscale image, I (with a domain Ω), at gray levels $\lambda_1 < \lambda_2$. Gray level causality holds if, for any $\mathbf{x} \in \Omega$, $L^{\lambda_2}(\mathbf{x}) = 1 \Rightarrow L^{\lambda_1}(\mathbf{x}) = 1$, or equivalently, $L^{\lambda_1}(\mathbf{x}) = 0 \Rightarrow L^{\lambda_2}(\mathbf{x}) = 0$. To establish gray level causality for inclusion filters, we first prove Lemma II.

Lemma II. Let S_1 and S_2 be two connected sets with holes $H_1^1, H_2^1, \ldots$ and $H_1^2, H_2^2, \ldots$, respectively. Let $S_1 \supset S_2$ and let $\mathbf{x}$ be a point, such that $\mathbf{x} \in F(S_2)$ and $\mathbf{x} \in H_i^1$, for some i. Then there exists H_j^2, for some j, such that $H_j^2 \supset H_i^1$.

Proof. Let any arbitrary $\mathbf{p} \in H_i^1$ and $\mathbf{p} \neq \mathbf{x}$. Given that $\mathbf{x} \in H_i^1$, there exists an unbroken path π from $\mathbf{x}$ to $\mathbf{p}$, such that $\pi \subset H_i^1$. Thus $H_i^1 \cap S_1 = \phi$ leads to:

$$\pi \cap S_1 = \phi. \tag{1}$$

Now $\mathbf{x} \in H_i^1 \Rightarrow \mathbf{x} \notin S_1$. Therefore $\mathbf{x} \notin S_2$ as $S_1 \supset S_2$. But it is given that $\mathbf{x} \in F(S_2)$, so there must exist a hole, H_j^2 of S_2, such that $\mathbf{x} \in H_j^2$. Let us now assume that $\mathbf{p} \in \overline{F(S_2)}$. Then the path, π from $\mathbf{x}$ to $\mathbf{p}$, must meet S_2, *i.e.*, $\pi \cap S_2 \neq \phi$. Since $S_1 \supset S_2$, we have the following:

$$\pi \cap S_1 \neq \phi. \tag{2}$$

Thus, (1) and (2) are contradictory, and our assumption that $\mathbf{p} \in \overline{F(S_2)}$ is false. Therefore $\mathbf{p} \in F(S_2)$. Now $\mathbf{p} \in H_i^1 \Rightarrow \mathbf{p} \notin S_1$. Then $S_1 \supset S_2 \Rightarrow \mathbf{p} \notin S_2$. Since $\mathbf{p} \in F(S_2)$ and $\mathbf{p} \notin S_2$, there must exist a hole H_l^2 of S_2, such that $\mathbf{p} \in H_l^2$. Let us now assume that $l \neq j$. Since S_2 surrounds both H_j^2 and H_l^2 and since $\mathbf{x} \in H_j^2$ and $\mathbf{p} \in H_l^2$, it follows that $\pi \cap S_2 \neq \phi$, which leads once again to (2). So $l = j$, as (2) is contradictory to (1). So we arrive at the result that for any arbitrary $\mathbf{p} \neq \mathbf{x}$, $\mathbf{p} \in H_i^1 \Rightarrow \mathbf{p} \in H_j^2$. Now we know that $\mathbf{x} \in H_i^1$ and $\mathbf{x} \in H_j^2$. Therefore we find that $\mathbf{p} \in H_i^1 \Rightarrow \mathbf{p} \in H_j^2, \forall \mathbf{p}$. So $H_j^2 \supset H_i^1$. <>

Now we are in a position to prove the preservation of gray level causality.

Proposition III (Preservation of gray level causality). $L_f^{\lambda_2}(\mathbf{x}) = 1 \Rightarrow L_f^{\lambda_1}(\mathbf{x}) = 1 \; \forall \lambda_1 < \lambda_2$.

Proof. Let the IS for the point $\mathbf{x}$ in the level set, L^{λ_2}, be $C_1^2, H_2^2, C_3^2 \ldots$, or $H_1^2, C_2^2, H_3^2 \ldots$, where C's and H's denote respectively foreground (=1) and background (=0) connected sets. Assuming gray level causality for the input image, for any foreground connected set, C_n^2 in L^{λ_2}, there exists a foreground connected set, C^1 in L^{λ_1}, such that $C^1 \supset C_n^2$. So by Lemma I we have $F(C^1) \supset F(C_n^2)$. Thus $\mathbf{x} \in F(C_n^2) \Rightarrow \mathbf{x} \in F(C^1)$. Therefore C^1 is present in the IS of $\mathbf{x}$ in L^{λ_1}, so there exists C_m^1, such that $C_m^1 = C^1$. Now, $L_f^{\lambda_2}(\mathbf{x}) = 1$; so there are two cases to consider:

Case (1): $T(F(C_1^2)) = 1$.

By the preceding argument there exists C_m^1 in the IS for $\mathbf{x}$, and $C_m^1 \supset C_1^2$. Now $\mathbf{x} \in C_1^2 \Rightarrow \mathbf{x} \in C_m^1$. So $m = 1$. Now by Lemma I we have $F(C_1^1) \supset F(C_1^2)$, and, by the increasing nature of T, we have: $1 = T(F(C_1^2)) \leq T(F(C_1^1))$, implying $T(F(C_1^1)) = 1$, and hence $L_f^{\lambda_1}(\mathbf{x}) = 1$.

Case (2): $T(F(H_n^2)) = 0$ and $T(F(C_{n+1}^2)) = 1$ for some $n \geq 1$. We have already shown that $\exists C_{m+1}^1 \supset C_{n+1}^2$. Two cases are possible here. In the first case, $m = 0$, *i.e.*, the IS of $\mathbf{x}$ in L^{λ_1} starts with C_1^1. Then, $1 = T(F(C_{n+1}^2)) \leq T(F(C_1^1))$, and thus, $L_f^{\lambda_1}(\mathbf{x}) = 1$. In the second case, $m > 0$, *i.e.*, there exists H_m^1 in the IS of $\mathbf{x}$ in L^{λ_1}. Then $F(H_m^1)$ is a hole in C_{m+1}^1 containing $\mathbf{x}$. Therefore by Lemma II, there exists a hole, H^2 in C_{n+1}^2, such that $H^2 \supset F(H_m^1)$ and $\mathbf{x} \in H^2$. Now, both $F(H_n^2)$ and H^2 are holes of C_{n+1}^2 such that $\mathbf{x} \in F(H_n^2)$ and $\mathbf{x} \in H^2$, which is not possible unless $H^2 = F(H_n^2)$. Therefore, the increasing nature of T again implies that $T(F(H_m^1)) \leq T(F(H_n^2)) = 0$ and $1 = T(F(C_{n+1}^2)) \leq T(F(C_{m+1}^1))$. This leads to $L_f^{\lambda_1}(\mathbf{x}) = 1$. <>

III. NUMERICAL RESULTS

In this section we show that inclusion filters can significantly enhance the performance of tracking moving targets. Automatic tracking of rolling leukocytes *in vivo* is pursued here to gain knowledge about the inflammatory process [9]. One of the characteristics of such intravital video sequences is the intensity reversal and contrast change of the leukocytes [9]. Thus the requirement of self-duality is crucial here. With a shape-size constrained snake, the snake finds the leukocyte boundary by minimizing an energy functional involving leukocyte-shape information and the image gradient [9]. The frames obtained via microscopy are often cluttered and noisy leading to error. So, before being processed by the tracker, the frames are inclusion filtered with the following increasing criterion:

$$T(F(C)) = \begin{cases} 1 \text{ if } F(C) \cap S \neq \phi, \\ 0, \text{otherwise}, \end{cases}$$

where $S = \{(x,y) : (x-\bar{x})^2 + (y-\bar{y})^2 \leq R^2\}$, $(\bar{x}, \bar{y})$ is the estimated leukocyte center and R is the radius of the leukocyte, known *a priori*. This infers that holes are filled only in the case where the holes do not have overlap with the prior 2-D cell shape. Figure 3(a) shows a part of an intravital video frame with leukocytes and the set S (inside the red contour). Figure 3(b) and 3(c) show respectively the gradient magnitudes without and with the application of inclusion filter.

One hundred tracking experiments of length 91 frames (duration 3 seconds) are used as the test dataset. These sequences are first tracked manually, then tracked with snake trackers with and without the application of the inclusion filter. As a performance measure, we consider position error as the distance (in microns) between the manually tracked leukocyte center and the tracker computed center. Another performance measure, *percentage of frames tracked*, is also considered. A frame is deemed "successfully tracked" if the position error for that frame is less than R. Thus, the percentage of frames tracked for a sequence is computed as the ratio of the total number of frames tracked in that sequence over 91. Figure 4 shows the comparison of mean position errors in tracking with and without the use of inclusion filters. Figure 5 shows the comparison of percentage of frames tracked with and without the use of the inclusion filter. Table 1 summarizes the comparative results by taking the mean measures over the one hundred sequences.

IV. CONCLUSIONS

In this paper, we define a self-dual connected operator by considering only 8-connectivity for foreground and background in a binary image. We also detail the extension to grayscale images. As an important application where self-duality of a filter is indispensable, we illustrate the enhancement of performance in tracking leukocytes *in vivo* through the use of the proposed filter.

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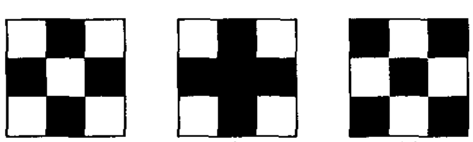

Figure 1. (a) Checkerboard L and $t(\overline{L})$. (b) $t(L)$. (c) $\overline{L}$ and $t(\overline{L})$.

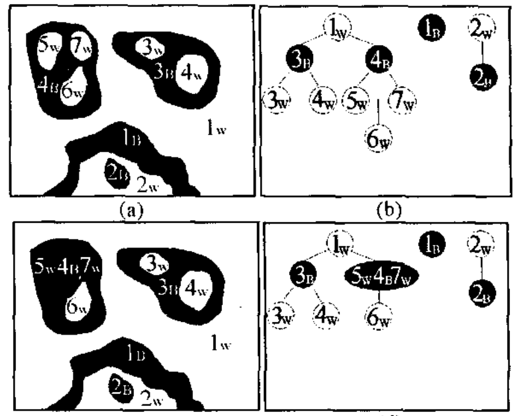

Figure 2. (a), (b) A binary image and the corresponding adjacency forest. (c), (d) Filtered image and the adjacency forest.

Figure 3. (a) Leukocytes, (b) gradient magnitude without inclusion filter and (c) gradient magnitude with inclusion filter.

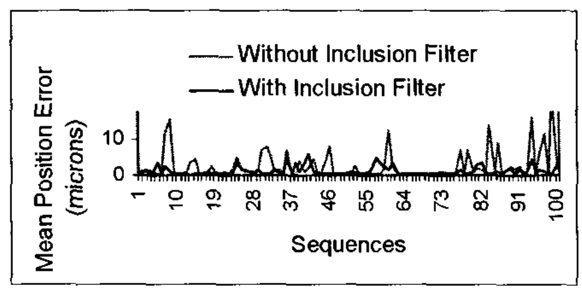

Figure 4. Mean position error with and without the inclusion filter.

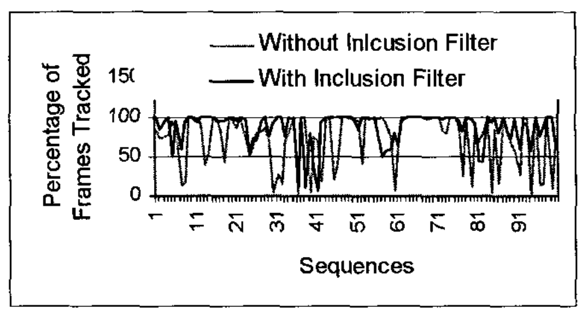

Figure 5. Percentage of frames tracked with and without the inclusion filter.

Table 1. Average performance measures.

Tracking Method	Average RMSE (microns)	Average Percentage of Frames Tracked
without Inclusion Filter	2.7	72
with Inclusion Filter	1.1	87