The Morphological Lomo Filter for Multiscale Image Processing

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Abstract

Locally monotonic (lomo) images are defined as root signals of a morphological lomo filter. The morphological approach allows a multidimensional generalization of local monotonicity. This generalization is well-motivated in that it retains the essential properties of one-dimensional (1-D) local monotonicity. Repeated application of the lomo filter produces a lomo root signal of a specified scale. By filtering at multiple scales, a locally monotonic scale-space can be created and used in multiscale image applications such as segmentation, tracking, content-based retrieval, and image coding. In contrast to existing linear and nonlinear scale-generating filters, the lomo filter has no spatial or graylevel bias and preserves edge localization through scale-space.

1. Local Monotonicity in 1-D

In one dimension, the concept of monotonicity is straightforward. A signal is monotonic over an interval (continuous or discrete) if it is either non-increasing or non-decreasing over that interval. A localized definition was introduced [1] in order to allow a measure of the local smoothness of a discrete signal, and can be extended to continuous signals easily. A 1-D signal is *locally monotonic* of degree n ("lomo-n") if and only if the signal is monotonic within every interval of length n.

Some basic properties of 1-D lomo-n signals are:

1. Plateaus between ascending and descending intervals: Between any increasing interval and any decreasing interval, there must exist a constant interval of length $\geq n$ (continuous) or $\geq n$-1(discrete) [1]. Any local minima or maxima is a member of adjacent ascending and descending intervals, and therefore is contained within a plateau. Fig. 1 shows a lomo-6 signal.

2. Separation of similar extrema: The distance between any two distinct (*i.e.* not contained within the same constant interval) local maxima is $\geq n$ (continuous) or $\geq$ n-1(discrete), and similarly for local minima. (Proof: A local minimum must exist between any two distinct local maxima, and that minimum must be contained within a plateau of length given by Property 1.)

3. Root signal of order-statistic filters: A lomo-n signal is a root of both the *open* filter and the *close* filter of constant-valued structuring elements, of length n (continuous) or n-1 (discrete), symmetric about the center*[2][3]. Therefore, the signal is also a root of any order-statistic filter that is bounded by the open and close filters, e.g. the *median* of length 2n (continuous) or length 2n-3 (discrete).

**Discrete* signals that are lomo-n with n odd (and thus n-1 is even) have a difficulty here. There is *not* a structuring element of even length that is symmetric about the center. However, these signals *are* roots of the median filter of length of length 2n-3.

4. Components of level-sets have a minimum 'scale': Each connected-component (both 1 and 0 connected objects) in each level-set (binary threshold) of a lomo-n object is at least large enough to enclose the morphological structuring element from Property 3. (Proof: Level-set objects must contain at least one local extremum, and Property 1 sets a lower bound on the level-set size of extrema.) Thus, there is a minimum size (*lomo scale*) associated with the level set components of a lomo-n signal. See Fig 1.

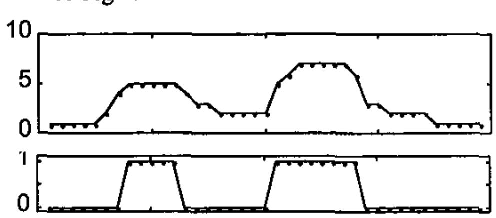

Figure 1. A discrete 1-D lomo-6 signal $f(x)$ and the level-set (or binary threshold) for $f(x) \geq 5$.

Each of the 1-D properties follows directly from the original definition based on non-increasing and non-decreasing segments. However, in higher dimensions,

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non-increasing and non-decreasing become ill-defined concepts. (Should the 2-D signal be non-increasing along *any* path of length n, any *straight* path, at least *one* straight path, *etc.*?) Therefore, it is of interest to see whether the 1-D definition can be stated in a more general, but equivalent, manner. In fact, with the 1-D case, any one of the four properties listed above is a necessary and sufficient condition for a signal to be identified as lomo-n. Thus, any of these can serve as the basic definition of local monotonicity. We choose among them the definition that most easily generalizes to multiple dimensions, while retaining the properties that are likely to be of greatest *utility* in application.

2. Multidimensional Local Monotonicity and the Morphological Lomo Filter

2.1 Previous Multidimensional Generalizations

Some attempts have been made to generalize local monotonicity to higher dimensions by way of the median filter. As mentioned in Property 3, a 1-D lomo-n signal is in fact a root of the symmetric median filter of the appropriate window or structuring element. In higher dimensions, a separable median filter has been proposed [4]. Separable median filters treat the horizontal and vertical directions separately, and therefore have inherent spatial bias in those directions. Instead, an isotropic definition is desired.

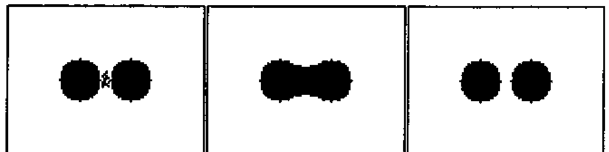
A spherically symmetric median in multiple dimensions is a possibility. However, when repeatedly applied to a signal, the median filter is susceptible to oscillations and generates streaking and blotching artifacts [5]. In addition, the median filter is rather cumbersome and inelegant in the continuous domain, both in theory and in computation. The basic concept of local monotonicity should have a simple definition in both continuous and discrete domains. As we will see, these goals can be satisfied through morphology.

2.2 Morphological Local Monotonicity

As mentioned above in Property 3, 1-D lomo signals are root signals of the morphological open filter and close filter simultaneously (of the appropriate structuring element). Let us define a multi-dimensional generalization of this definition of local monotonicity:

Definition 1: A signal is **strict-sense morphological lomo-n** if and only if it is a root signal of the morphological filters open and close (simultaneously) where the structuring element is spherically symmetric of radius $r(n)$ and of constant value. The structuring element is centered on a single point and contains all points whose distance from the center is less than or equal to r. For continuous domain signals this implies $r = n/2$, and for discrete signals $r = (n-2)/2$, in agreement with the 1-D case.

In 1-D a single pass of the open-close (or close-open) filter creates a root signal of both the open and close filters. However, the choice of applying one filter or the other to a non-lomo signal would impose a graylevel bias (the open-close filter is biased towards lower intensity and the close-open filter towards higher intensity). In higher dimensions, even this procedure breaks down, and the open-close filter produces a signal that is a root of the close filter but not necessarily of the open filter (See Fig. 2 for an example). A similar statement can be made about the close-open filter. Thus, the procedure for obtaining a lomo signal from a non-lomo signal is unclear.



(a) (b) (c)

Figure 2. A simple 2-D example of a saddlepoint situation. From left to right: (a) original test image, (b) after the open (or close-open) filter of structuring element equal in size to the circular object ($r = 10$), (c) after the close (or open-close) filter.

Furthermore, signals that are roots of both open and close may be too restrictive a set for useful applications. For example, suppose a non-lomo 2-D image is given, and it is desired to transform it into a lomo signal. For convenience, let us define a *regular lomo region* as a region the size of the structuring element that is neither altered by the open filter nor by the close filter. Consider the original 2-D image in Fig. 2a. When the original is filtered (separately) by the open filter (Fig. 2b) and close filter (Fig. 2c), the regular lomo regions are those pixels left unchanged. However, by the above definition, there is no lomo-n signal that leaves all these pixels unchanged.

Thus, creating a strict-sense morphologically lomo signal from a non-lomo signal (such as in Fig. 2a) may require that regular lomo regions be altered. It is desired, however, that in creating a lomo signal from a non-lomo signal, all regular lomo regions be left unaltered. This constraint maintains fidelity to the original signal (and can be exploited for computational efficiency). As we will see, this objective can be met with a slightly modified morphological filter.

Definition 2: A M-dimensional signal f is **morphological lomo-n** if and only if it is a root signal of the M-dimensional lomo filter of scale n. The **lomo filter** is defined as the linear combination of open and close filters of spherically symmetric, constant-valued structuring element k (of radius $r(n)$):

$$f \leftarrow \frac{f \circ k + f \bullet k}{2}, \tag{1}$$

where the structuring element and relationship between n and $r(n)$ are the same as in Definition 1. This filter has been considered as a morphological approximation to the median [6]. However, the operator fails to be idempotent. Therefore, (1) is written as an iterate, and the filter is repeatedly applied until convergence.

This definition is equivalent to other 1-D definitions. However, the introduction of the lomo filter has significant advantages in higher dimensions. First, iterative application of the filter converges to a lomo root signal. Hence, a procedure for obtaining a lomo signal from a non-lomo one is achieved. Furthermore, all regular lomo regions are unaltered by the filter. The filter contains no directional bias, and applies easily to both discrete and continuous signals. Similar to the median filter, the lomo filter is self-dual and thus possesses no graylevel bias [7].

The four 1-D properties mentioned above become modified by both Definitions 1 and 2 in higher dimensions. By either definition, Property 1 becomes the following: Any local extremum is contained within a constant region such that the structuring element fits within that region, that is a constant regular lomo region. This gives these constant regions a minimum size (radius) corresponding to the lomo scale.

Property 2 is unaltered by Definition 1. However, Definition 2 allows the existence of saddlepoints between similar extrema. These extrema can be closer together than a single structuring element width. The treatment of these saddlepoints marks the fundamental difference between Definitions 1 and 2 and will be analyzed in more detail below.

Property 3 is also altered, as the relationships between the order statistic filters becomes more complex. Open and close no longer serve as bounds for the median in 2+ dimensions. Thus, the roots of the lomo filter are not necessarily median roots. In contrast to signals satisfying Definition 1, signals satisfying Definition 2 are not necessarily roots of both open and close separately. Again, the difference is the treatment of saddlepoints. For example, the signal in Fig. 2a becomes the (Definition 2) lomo signal in Fig. 3 (upon repeated application of the lomo filter). It contains a *saddlepoint region* between two local maxima (and two local minima), *i.e.* a portion of the signal which is a root of the lomo filter, but not a root of either the open filter or the close filter.

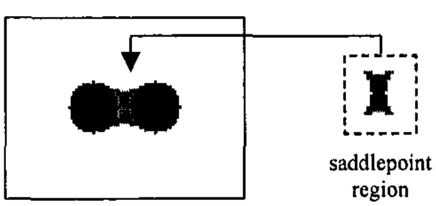


Figure 3. 2-D signal of Fig. 2(a) after lomo filtering (root signal reached after a single pass). The gray region is a saddlepoint region. All other points are regular lomo points.

Property 4 is retained in higher dimensions by both definitions. For Definition 1, the 1-D proof can be generalized immediately. For Definition 2, saddlepoint regions must be considered. However, there can be no level-set object contained entirely within a saddlepoint region, because such regions contain no local extrema. Thus, all level-set points in saddle regions are connected to regular lomo regions that satisfy Property 4.

Definition 2 is less restrictive than Definition 1, and the root set of Definition 2 contains that of Definition 1. If the original signal contains saddlepoint regions (as often occurs in practice), there is no clear filtering procedure for eliminating them and producing a root signal of open and close simultaneously, as Definition 1 requires. Definition 2, in contrast, does provide a filtering procedure, but allows saddlepoint regions (excluded by Definition 1) to remain.

2.3 Alternative Lomo Filters

While the iterate given by (1) satisfies our need for a filter that generates morphological lomo signals, it is noteworthy that this filter is not unique. For example, a root signal of (1) is also a root signal of both

$$f \leftarrow \frac{(f \circ k) \bullet k + (f \bullet k) \circ k}{2}, \tag{2}$$

and

$$f \leftarrow \frac{((f \circ k) \bullet k) \circ k + ((f \bullet k) \circ k) \bullet k}{2}. \tag{3}$$

Here we present a brief outline of the proof. Assume f is a root signal of (1):

$$f \circ k + f \bullet k = 2f. \tag{4}$$

Then,

$$f \bullet k = f + (f - f \circ k) = f + \rho_\circ \tag{5}$$

where $\rho_\circ = f - f \circ k$ is the open residue. Opening (5) gives

$$(f \bullet k) \circ k = (f + \rho_\circ) \circ k. \tag{6}$$

Since the addition of the open residue of f to a signal f does not affect subsequent open filtering, we obtain

$$(f + \rho_\circ) \circ k = f \circ k. \tag{7}$$

Combining (6) and (7), and demonstrating the dual case for the close operation ($(f \circ k) \bullet k = (f - \rho_\bullet) \bullet k = f \bullet k$), the filter in (2) reduces to the filter in (1). The same property extends this proof to include the filter in (3).

This implies that each of these lomo filters may serve as an equivalent *definition* of local monotonicity of a given scale by sharing the same set of root signals. However, it should be noted that iterative application of each of these filters leads to different lomo roots within that root set. Alternative lomo filters may be advantageous. For example, (1) attenuates an image impulse at a geometric convergence rate, while (2) eliminates the impulse in a single iteration.

3. Lomo Scale-Space

A signal scale-space is a series of representations that vary from fine to coarse. The proposed lomo filter possesses many of the same characteristics of other morphological scale-generating filters, such as edge localization. However, because the lomo filter has no graylevel bias, the results of lomo filtering tend to remain more faithful to the original signal.

In generating a lomo scale-space, each image of increasing scale is formed by iteratively applying the lomo filter to the previous image in scale-space, similar to morphological alternating sequential filtering [8]. This generation technique allows noise and small features to be removed effectively as scale increases. Three selected levels of the lomo scale-space for the original image of Fig. 4 are shown in Fig. 5.

Figure 4. Original 256 x 256 pixel image with 256 graylevels used in generating the scale-space of Fig. 5.

Table. Mean Squared Error (w.r.t. the original Fig. 4) of three scales from three morphological scale-spaces: close-open and open-close alternating sequential scale-spaces and the lomo scale-space, using 256 graylevels and circular structuring elements of specified radii.

	radius 1 pixel	radius 2 pixels	radius 4 pixels
close-open	**29.3**	**63.9**	**153.4**
open-close	**29.5**	**76.9**	**191.3**
lomo	**27.1**	**59.8**	**113.7**

Figure 5. Selected levels of the lomo scale-space derived from Fig. 5. From top, circular structuring elements of radii 1,2, and 4 pixels.

The self-dual lomo scale-space outperforms its graylevel-biased morphological counterparts. The lomo scale-space therefore represents a viable alternative to previous scale-spaces, by its fidelity to the original image, lack of intensity bias, and well-defined scale properties. Quantitatively, the Table gives the mean squared error (MSE) of three scale-space levels measured from the original image of Fig. 4 using morphological alternating sequential scale-spaces and the lomo scale-space generated by (1).

4. Applications and Further Research

The motivation behind lomo filtering is the desire to design a scale-generating pre-filter for multiscale image processing tasks. Applications may include multiscale object tracking, segmentation, content-based retrieval, and other coarse-to-fine search processes, as well as multiscale image/video coding.

For example, edge detection and ultimately segmentation could be performed using the following technique. A discrete approximation of the second derivative of a 1-D lomo-n signal can be expressed morphologically using the linear combination of dilation and erosion operations:

$$s(x) = \frac{f(x) \oplus k(x) + f(x) \ominus k(x)}{2}. \qquad (8)$$

The difference signal between the lomo signal $f(x)$ and this *midrange* [7] signal $s(x)$ is identical to the common discrete approximation formula given by

$$f''(x) \cong \frac{f(x+\Delta) - 2f(x) + f(x-\Delta)}{2}, \qquad (9)$$

for any $\Delta \leq n/2$. Thus, zero-crossings in the difference signal $s(x) - f(x)$ are akin to zero-crossings in the second derivative of $f(x)$.

This morphological edge detection could be extended to 2-D as an alternative to the Laplacian of Gaussian. Rather than applying the Laplacian operator within linear scale-space, this morphological second derivative approximation is used within a morphological scale-space. In this manner, a scaled edge detector is properly matched with a similarly scaled signal. An example of this zero-crossing edge detection is shown in Fig. 6. This technique is one of many possibilities for taking advantage of the well-defined scale properties of lomo signals.

Areas requiring further research include the convergence rate of the lomo filter and the characteristics of the alternative lomo filters. Also, a study of sampling conditions for the lomo filter may show that a lomo scale-space pyramid can be created and used in a pyramidal image analysis.

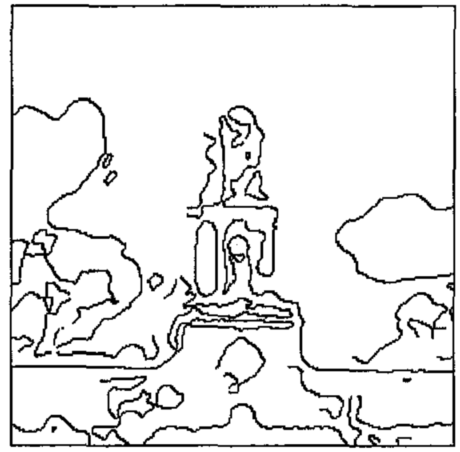


Figure 6. Zero-crossings of the (thresholded) difference image $s(x) - f(x)$, where $f(x)$ is the radius-4 lomo signal of Fig. 5, and $s(x)$ is given by (8).

5. References

[1] S.G. Tyan, "Median filtering: deterministic properties," in *Topics in Applied Physics, Two-dimensional Signal Processing II*, T.S. Huang, Ed. Berlin, Germany: Springer-Verlag, vol.43, pp.197-217, 1981.

[2] R. M. Haralick and L. G. Shapiro, *Computer and Robot Vision*, vol. I, New York: Addison-Wesley, pp. 215-218, 1992.

[3] P. Maragos and R. Shaefer, "Morphological filters-part II: their relations to median, order statistic, and stack filters," *IEEE Trans. Acoust., Speech, and Signal Process.*, vol. ASSP-35, pp. 1170-1184, 1987.

[4] P.M. Narendra, "A separable median filter for image noise smoothing," *IEEE Trans. on Pattern Analysis and Machine Vision*, vol. PAMI-3, no.1, pp. 20-29, 1981.

[5] A. C. Bovik , "Streaking in median filtered images," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-35, pp. 493-503, 1987.

[6] M. A. Schultze and J. A. Pearce, "Linear combinations of morphological operators: the midrange, pseudomedian, and LOCO filters," *IEEE Conference on Acoustics, Speech, and Signal Processing*, ICASSP-93, vol. 5, pp. 57-60, 1993.

[7] H. J. A. M. Heijmans, "Construction of self-dual morphological operators and modifications of the median," *Proc. IEEE Int. Conf. On Image Processing*, ICIP-94, vol.2, pp. 492-496, 1994.

[8] J. A. Bangham, P. D. Ling, and R. Harvey, "Scale-space from nonlinear filters," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 18, no. 5, pp. 520-527, 1996.