A PYRAMIDAL ALGORITHM FOR AREA MORPHOLOGY

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ABSTRACT

A fast algorithm that approximates area morphological filters is presented. Area morphological filters enable the elimination of connected components within image level sets according to area alone. To date, area morphological filters have been limited in application due to the high computational expense involved with connected component analysis. The pyramidal implementation described here dramatically reduces the computational cost by simultaneously creating a marker image and performing reconstruction in a coarse-to-fine manner. The pyramidal algorithm reduces the computational cost by three orders of magnitude for standard-sized grayscale images. Image examples and an analysis of the computational expense are provided in the paper.

1. INTRODUCTION

Area morphology is a departure from traditional morphology. In the area morphological approach, the grayscale image is essentially decomposed into a number of binary (thresholded) image representations – *level sets*. Then, self-connected regions within these binary images, the *connected components*, can be preserved or removed (complemented) in their entirety based upon a minimum area parameter. The net effect of such processing is that objects of insufficient area can be removed, yielding a truly scalable nonlinear filter. With traditional morphology, these connected components would be modified based on the inscription with a structuring element of specified shape and size. Thus, at the borders of the connected components, the edges are distorted according to the shape of the structuring element.

The edge localization, Euclidean invariance, and causality properties of scale spaces generated by area morphology have been utilized in recent applications such as document page segmentation [2], image reconstruction [3], and image classification [1]. The major drawback of area morphology is the immense computational cost associated with the connected component analysis. In order to use area morphology in time critical applications, we have developed algorithms that can reduce the processing time for a typical area morphology operation by a factor of 1000.

The paper provides a brief summary of area morphological operations, describes the fast algorithms and provides results that compare the new pyramidal approach with the standard implementation and with other recently introduced fast algorithms.

2. AREA MORPHOLOGY

We may consider the *area open* operation as the process of removing bright objects that do not meet the specified minimum area. Likewise, the *area close* operator removes dark objects of insufficient area. The objects in this case are connected components within the *level sets* of the image. For an image with discrete domain $\mathbf{D} \subset \mathbf{Z}^2$ and image location $p \in \mathbf{D}$, level set $\mathbf{S}_l$ at level l, where $l \in [0, L-1]$, $l \in \mathbf{Z}$, is defined by $S_l(p) = 1$ if $I(p) \geq l$, and $S_l(p) = 0$ otherwise. Within a level set $\mathbf{S}_l$, the connected component $\mathbf{C}_{S_l}(p)$ at p is given by $\mathbf{C}_{\mathbf{S}_l}(p) = \{q : \exists P_{I \geq l}(p,q)\}$, where $P_{I \geq l}(p,q)$ is an unbroken path between image locations p and q for which $S_l(\cdot) = 1$ (hence $I(\cdot) \geq l$). The neighboring elements in such a path can be defined by 4 or 8 connectivity. (We assume 4-connectivity in this paper.)

Given the definitions of level sets and connected components within the level sets, we can define the area open and area close operators. First, we define the operations on individual level sets and then for a grayscale image. For level set $\mathbf{S}_l$, the area open operation is given by

$$\mathbf{S}_l \circ (a) = \{p : \exists \left|\mathbf{C}_{\mathbf{S}_l}(p)\right| \geq a\}. \qquad (1)$$

where a is the minimum area (in terms of the number of pixels). Similarly, an *area close* operation on a level set is defined by

$$\mathbf{S}_l \bullet (a) = \{p : \exists \left|\mathbf{C}_{\tilde{\mathbf{S}}_l}(p)\right| \geq a\}. \qquad (2)$$

In case of area close, the cardinality term $\left|\mathbf{C}_{\tilde{\mathbf{S}}_l}(p)\right|$ is defined on the complement of level set $\mathbf{S}_l$, that is, where $S_l(p) = 0$.

For grayscale images, we can define area open and area close by stacking the processed level sets. The

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reconstructed area-opened image at scale a is given by $\mathbf{I} \circ (a) = \sum_{l=0}^{L-1} \mathbf{S}_l \circ (a)$, and the area-closed image is given by $\mathbf{I} \bullet (a) = \sum_{l=0}^{L-1} \mathbf{S}_l \bullet (a)$. In this standard implementation, a connected component labeling procedure is enacted on each level set, and then the connected components that do not meet the minimum area are complemented. Given an image with L intensities and N pixels, this $O(LN)$ operation is extremely expensive computationally.

3. FAST AREA MORPHOLOGY ALGORITHMS

As area close is the same as applying area open to a level set complement, we treat only the case of area open here. Previously introduced fast algorithms for the area open filter essentially use the traditional open filter as a starting point [4]. Then, any connected components within the level sets that partially "survive" the open operation are fully reconstructed. Vincent [4] first explored this approach, calling the original image a *mask* image **I**, the opened image a *marker* image **M**, and the final product a grayscale *reconstruction* **R**. So, the problem of designing a fast area open operation can be divided into marker image creation and reconstruction.

3.1. Marker image creation by translation variant pyramids

The pyramidal algorithm for area morphology is based on the notion that we can successively eliminate small image regions by creating coarser pyramid levels. Then, the surviving regions of sufficient area can be reconstructed in a coarse-to-fine manner. For an area open operation in particular, we can use an erosion pyramid (erosion and downsampling) to eliminate small objects and then recreate the connected components at the original image resolution by successively dilating and upsampling. Consider a pyramid **P** with levels $m \in \{0,1,\ldots,M\}$, where pyramid level $\mathbf{P}_0 = \mathbf{I}$. To create a marker image for an area open with area $a = 4^M$, we first create the M+1 level erosion pyramid. Then we successively dilate from coarse-to-fine in order to compute the marker image **M** at the original resolution of N pixels. These two steps, analysis and synthesis, are given by:

<u>Analysis step</u>. Let $\mathbf{P}_0 = \mathbf{I}$. For levels $m > 0$,

$$\mathbf{P}_m = (\mathbf{P}_{m-1} \ominus \mathbf{F}_0) \downarrow \tag{3}$$

where $\mathbf{F}_0$ is the 2x2 structuring element with the origin in the upper left. The downsampling operation, denoted by $\downarrow$, is the injection operator, where the upper-left value of each 2x2 image subsection is sampled in creating an image that is half as wide and half as high as that of the previous pyramid level. It is important to note that at level M, the value of the pixels will represent the highest level set for which a connected component of size 2^M x 2^M (area a) exists in the original image.

<u>Synthesis step</u>. To recreate the connected components, we use the following relationship, starting at level M-1:

$$\mathbf{P}_m = [(\mathbf{P}_{m+1}) \uparrow] \oplus \mathbf{F}_0 \tag{4}$$

where the upsampling operator $\uparrow$ simply injects the pyramid values in a matrix of zeros that is twice as wide and twice as high. The dilation step serves as a prolongation operation.

After synthesis, $\mathbf{P}_0$ is used as the marker **M**.

3.2. Marker image creation by translation invariant pyramids

The main drawback of the previously described pyramidal algorithm is translation variance. Small translations can affect the outcome of preserving/eliminating connected components in the marker image. This shortcoming can be alleviated by redundancy. The translation invariant method uses a series of erosions in which the structuring elements themselves are translated. Similarly, in the synthesis step, a group of dilations are used and the results are taken in union.

In the case of the translation invariant pyramid, let $\mathbf{P}_m$ define a set of images indexed by an m-dimensional vector $(i_1, i_2, \ldots, i_m)$, where $\mathbf{P}_0 = \mathbf{I}$ initially. For each erosion in the translation variant pyramid, we will construct four erosions in the translation invariant method.

<u>Analysis step</u>. At level m, for each $j \in \{0,1,2,3\}$

$$\mathbf{P}_m(i_1, \ldots, i_{m-1}, j) = (\mathbf{P}_{m-1} \ominus \mathbf{F}_j) \downarrow \tag{5}$$

where $\mathbf{F}_j$ refers to the set of 2x2 structuring elements with origins at (0,0), (-1,0), (0,-1), and (-1,-1), relative to the upper left pixel.

<u>Synthesis step</u>. We recreate the higher resolution levels by

$$\mathbf{P}_m(i_1, \ldots, i_m) = \bigcup_{j=0}^{3} [\mathbf{P}_{m+1}(i_1, \ldots, i_m, j) \uparrow] \oplus \mathbf{F}_j . \tag{6}$$

So, at level m = 0, we union four dilated images, and the marker image **M** is achieved. We will see that the true advantage of the pyramidal approach is not marker image creation, but a full algorithm that computes both the marker and the reconstruction simultaneously.

3.3 Reconstruction by geodesic dilation

Reconstruction is the process of taking a partial connected component and recreating the entire connected component, based on the intensities in the input image **I**. We can reconstruct the connected components by selectively dilating these components (one pixel at a time):

$$R_t(p) = \min\{(\mathbf{R}_{t-1} \oplus \mathbf{K}^+)(p), I(p)\} \tag{7}$$

where $p \in \mathbf{D}$, $\mathbf{R}_0$ = **M**, and $\mathbf{K}^+$ is a 3x3 cross-shaped structuring element with the origin at the center. The update in (7) will stabilize when each of the marked connected components is reconstructed. Geodesic dilation and a fast queue-based geodesic dilation were used by Vincent [4] for reconstruction. In our pyramidal approach, we apply a limited number of geodesic dilations at each pyramid level.

3.4 The full pyramidal algorithm: marker image creation and reconstruction

The salient advantage of the pyramidal algorithm is that reconstruction can be performed simultaneously with marker image creation. In the synthesis step for either the translation variant or invariant pyramid, a small number of geodesic dilations (7) can be implemented on each upsampled pyramid level after applying (4) or (6). The multiresolution reconstruction is expeditious. One geodesic dilation at level m takes care of 4^m geodesic dilations at level 0 (at the original resolution).

4. COMPLEXITY ANALYSIS

As area open and area close have the same complexity, we will examine only the complexity of the area open operation and the associated fast algorithms. The standard method of implementing an area open operation involves connected component analysis on each image level set. For the sake of a simplified analysis, let us assume that a comparison operation, an addition/subtraction operation, a Boolean operation and an assignment operation each represent one operation on a standard serial computer architecture. For the connected component analysis, let us also assume that 4-connectivity (the least expensive option) is employed. Under these assumptions, we need 22 operations for each level set element to achieve connected component analysis. Then, we need 2 more operations per level set element to achieve the compare/complement step in the area open process. So, a total of $24LN$ operations are required for an image with N pixels and L discrete intensity levels.

For the fast algorithms, we must break down the computational cost into the cost contributed by marker creation and the cost contributed by reconstruction. A standard open filter will require $2aN$ comparisons (aN for erosion and aN for dilation) to compute the marker for area a. The expense of the analysis step in translation variant pyramidal marker creation is $N/4^m$ operations at level m, since only one of four erosions needs to be computed within each pyramid level. In the synthesis step, the expense is $4N/4^m$ to recreate level m. So, for the translation variant pyramid, we have $\sum_{m=0}^{M-1} \frac{5N}{4^m}$ operations, which is $O(N)$. Here, the number of pyramid levels, M, is equal to $\log_4 a$. The translation invariant pyramid, on the other hand, needs $4N$ dilations and $4N$ erosions at each pyramid level. The $8MN$ operations lead to an overall complexity that is $O(N \log_4 a)$.

For the reconstruction, we examine three possibilities: reconstruction by geodesic dilation, reconstruction by Vincent's queue-based algorithm [4] and combined marker creation/reconstruction using the new pyramidal algorithm. Each step in geodesic dilation requires $5N$ comparisons (4 for dilation and 1 for the minimum operation). Iteration in geodesic dilation continues for T steps, where T is bounded by the maximum geodesic distance between an element on the boundary of a marker connected component and the corresponding boundary element in the reconstruction. If we assume $\max(T) \approx N^{1/2}$, then geodesic dilation has a complexity that is $O(N^{3/2})$. The queue-based algorithm introduced in [4] needs $8N$ operation for initialization and $16N$ operations for each pixel in the queue. If a fraction k of the total pixels are placed on the queue, then this algorithm requires $(8 + 16k)N$ operations, which is essentially $O(N)$.

The combination of marker creation and reconstruction in the pyramidal framework is quite advantageous computationally. Essentially, the pyramidal method utilizes c geodesic dilations at each pyramid level, while creating the marker image. So, $5c$ operations are performed on each pyramid level and each element for the geodesic dilation. One geodesic dilation at level m accomplishes 4^m geodesic dilations at level 0 (the original image resolution). Therefore, the total cost of marker creation and reconstruction for the translation variant approach is $\sum_{m=0}^{M-1} \frac{25cN}{4^m}$. Using the translation invariant approach, the cost is increased to $40cMN$ operations. As the results demonstrate, the pyramidal approach can lead to a dramatic reduction in computing times for the area open and area close operations.

A summary of the complexity analysis for the area open operation is found in Table 1.

5. RESULTS AND CONCLUSIONS

The experiments reveal that the reconstruction step is the most burdensome portion of implementing a fast area morphology operation. The pyramidal algorithm provides a slight improvement over the standard morphological filter in terms of marker creation. However, the strength of the pyramidal approach is not fully demonstrated until the complete area operation, combining marker creation and reconstruction, is examined.

In Table 1, we see that pyramidal marker image creation and reconstruction together are linear w.r.t. N, the number of image pixels. The complete pyramidal approach is over 1500 times faster than the standard area open implementation!

Figure 1 gives an example of area open and fast area open results. The area open operation provides an effective method to remove glints and specularities in the image of neoplastic cells in Fig. 1(a). The subjective quality of the area open operation in Fig. 1(b) is matched by the fast algorithm results shown in Fig. 1(c-e). So, the pyramidal algorithm is able to process the image with a dramatic improvement in speed at equivalent quality.

In summary, we have presented a fast pyramid-based implementation that approximates area morphological filters. The filters eliminate objects in the image level sets of insufficient area. Therefore, area morphology should prove useful in several image processing problems such as segmentation and classification. In future work, we will provide an analysis of quality differences between the pyramidal implementation and the standard implementation based on connected component analysis.

6. REFERENCES

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Table 1: Computational expense for area open. Key to variables used: $a \equiv$ area, $N \equiv$ number of images pixels, $M \equiv$ number of pyramid levels where area $a = 4^M$ (a = 64 here), $T \equiv$ maximum geodesic distance between the unreconstructed and reconstructed connected component boundaries, $k \equiv$ fraction of pixels changed in the reconstruction process, $L \equiv$ number of intensity levels, and $c \equiv$ number of dilations for each pyramidal level (for reconstruction) – c = 4 here.

	Standard Area Open	**Open w/ Geodesic Dilation**	**Variant Pyr. with Queue Recon.**	**Full (Trans. Variant) Pyr. Alg.**
No. Operations	$24LN$	$(2a+5T)N$	$\sum_{m=0}^{M-1} \frac{5N}{4^m} + (8+16k)N$	$\sum_{m=0}^{M-1} \frac{25cN}{4^m}$
Complexity	$O(LN)$	$O((a+N^{1/2})N)$	$O(N)$	$O(N)$
Time on 128x128	102.00 s	3.46 s	8.25 s	0.71 s
Time on 256x256	1437.30 s	21.47 s	38.68 s	1.32 s
Time on 512x512	6046.60 s	123.86 s	150.06 s	3.68 s

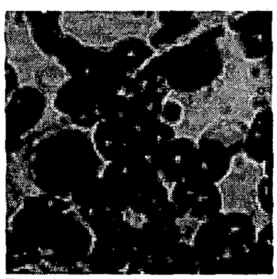
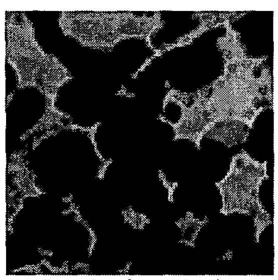
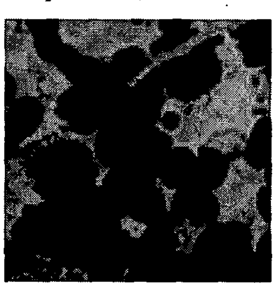


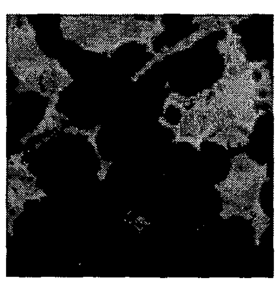
Fig. 1: (a) Original image of neoplastic cells.



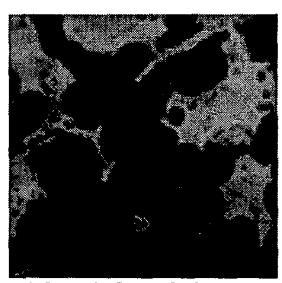
(b) Result of standard area open w/ a = 64.



(c) Result using open for marker image and geodesic dilation for reconstruction (a = 64).



(d) Result using trans. variant pyramid w/ queue-based reconstruction (a = 64).



(e) Result from full trans. variant pyramid (a = 64).