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Morphological scale-space in image processing

Joseph H. Bosworth and Scott T. Acton *

Department of Electrical Engineering, University of Virginia, Charlottesville, VA 22904, USA

Abstract

The purpose of this paper is twofold. First, we provide an extensive review of the state-of-the-art in scale-space generation techniques for image processing, including linear methods, diffusion-based methods, and emphasizing morphological methods. Then, we introduce a new morphological approach to scale-space, called the *lomo* scale-space. The technique introduces a novel two-dimensional generalization of the concept of locally monotonic (lomo) signals. The lomo scale-space is a sequence of locally monotonic image representations where the scale is specified by the spatial extent or degree of local monotonicity. The morphological process used to generate the lomo scale-space retains many desirable properties of other morphological methods, such as edge localization and smoothing of extrema. In contrast to previous morphological scale-space methods, the filters employed here are self-dual, and thus do not induce a gray-level bias into the scaled signal representations. The scale-space methods reviewed and introduced in this paper are applicable to several multiscale image processing tasks such as segmentation, object-based image compression, content-based retrieval, and video tracking.

Keywords: Non-linear filters; Scale-space; Morphology

1. Introduction

A scale-space is a multiscale representation of a signal (from coarse to fine) satisfying certain properties. For example, the scale-space representation of an image is a series of successively simplified images. A scale-space is obtained by a particular filtering procedure that serves to remove detail and to reduce the information content of the image, while retaining the essential features. The choice of the scale-space filtering technique depends on the application, but is restricted by the requirements of some basic scale properties.

* Corresponding author.
E-mail address: acton@virginia.edu (S.T. Acton).

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These properties include the concepts of fidelity, causality [1], Euclidean invariance [2], and continuity [3], and each will be treated in this paper.

The utility of the scale-space concept lies in its simplification of signal information for signal processing and analysis tasks. Here, our concentration will be on two-dimensional (2-D) images, where nearly every image analysis task requires the selection of at least one scale parameter. For example, in video tracking or object recognition the target may be of a given scale, and it is desired to simplify the image sequence by eradicating extraneous small-scale detail and noise.

Often, the simplification of an image is needed solely for computational efficiency or data compression, as an increase in scale is accompanied by a reduction of information. In addition to simplifying the content of a signal at a given scale, the relationships between *different* scales can be used in extracting important information for image analysis and computer vision tasks. Many multiscale image processing techniques include an interaction between different scales, such as coarse-to-fine searches. Thus, the creation of the scale-space itself is an essential initial step in these image processing applications. Morel and Solimini [2] go so far as to say "the only basis parameter in computer vision algorithms is 'scale' . . . computer vision algorithms are always stated with a 'multiscale' formulation."

In this paper, we first give an overview of scale-space theory, including the original linear, or Gaussian scale-space. Then, with the basic axioms of classical scale-space theory established, we review recent research in the non-linear extension of scale-space theory, with an emphasis on morphological techniques. Finally, a new morphological scale-space, called the *lomo scale-space*, is introduced. This new scale-space retains the desirable properties of previous morphological scale-spaces, while avoiding the drawbacks of previous morphological techniques. The proposed lomo scale-space generalizes to higher dimensions, and therefore is of use in image and video processing.

2. Scale-space requirements

Here we introduce the notation of a scale parameter s, and a scale-space $F(\mathbf{x}, s)$ obtained from the original signal $f(\mathbf{x})$, a real-valued function of the time (for 1-D signals) or spatial position (for 2-D images). For continuous-domain signals, $f(\mathbf{x}): R^n \to R$ and $F(\mathbf{x}, s): R^n \times R^+ \to R$. We will also consider discrete-domain signals, using the same notation $F(\mathbf{x}, s)$, though $\mathbf{x}$ will be restricted to a subset of integer-valued indices. For notational convenience, we write $F(s)$ instead of $F(\mathbf{x}, s)$ when $\mathbf{x}$ is implied. Note that the scale parameter is only allowed to take on non-negative values, where the zero scale indicates the original signal.

For a multiscale signal representation to be accepted as a scale-space, it must hold certain properties. These properties are all intuitive, and they are all based on the concept of successive simplifications of the signal as scale is increased. The first of these properties is *fidelity*. This is simply the requirement that as the scale parameter approaches zero, the scale-space signal approaches the original signal. The fidelity requirement can be written as

$$\lim_{s \to 0} F(\mathbf{x}, s) = f(\mathbf{x}). \tag{1}$$

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We have not yet explicitly defined the scale parameter s itself, and indeed it depends on the particular filtering procedure used in creating the scale-space. However, the scale parameter does always correspond in some sense to a spatial scale (or perhaps a temporal scale if we are dealing with a one-dimensional speech signal, for example). As an example of a scale parameter in image processing, we mention the Gaussian scale-space in which each scale is generated by convolving the original image by a symmetric Gaussian kernel. Here, the standard deviation of the Gaussian kernel can be considered the scale parameter.

Another property required for a scale-space representation is *causality*, that is, each level of the scale-space $F(a)$ depends solely on $F(b)$ if $a > b$. Thus, the coarser scales are derived from the finer scales, but not vice versa. Related to the causality property is the *monotone* property of *signal features*. The monotone property states that the number of signal features should monotonically decrease as the scale parameter increases. Often in the literature, the term “causality” is generically used in describing the monotone property of features [1]. However, here we wish to distinguish the dependence through scale of features from that of the image itself, and we exclusively use the term “monotone” for features and “causality” for the signals themselves. Thus, in keeping with our goal of signal simplification, features should only disappear and not be created as scale increases. As with the scale parameter, the definition of the signal features depends on the specific filtering procedure used and will be discussed for each individual scale-space described below.

A feature can be any point, alone or one of a set of points, which represents some useful information about a signal. Features should represent some simplification and reduction of information that gleans the essential information from the scale-space representation. By extracting features at different scales and following them through the scale-space, signal processing tasks become simplified and more efficient. Typical image processing tasks that utilize feature extraction are video tracking, image correspondence problems (such as in stereo vision), image segmentation, content-based retrieval (CBR), and other coarse-to-fine searches. Common feature definitions include the zero-crossings in the second derivative or Laplacian, local signal extrema, or edges as defined by a particular edge detector.

In addition to the monotone property of features, scale-space features are required to exhibit a *continuity* property. Not only should the number of features decrease with increasing scale, but also those features that remain at a higher scale should correspond to features existing at a lower scale. Note that this correspondence does not restrict feature positions at a higher scale to be a subset of those at lower scales. Features are allowed to “drift” spatially through scale along continuous paths. However, if the higher-scale feature positions are indeed a subset of those at lower scale, it is said that the scale-space possesses *strong causality* [2].

As an example of features and their continuity within a scale-space, Fig. 1 shows a Gaussian scale-space of a discrete one-dimensional signal, where features are defined as local extrema. The paths traced by these features through scale-space create a plot (such as in Fig. 2 for the one-dimensional example of Fig. 1) known as the scale-space *fingerprint* [4]. It serves as an aid in visualization of the correspondence and possible movement of features as scale increases. Often, as is the case for their likeness—the human fingerprint, useful information identifying a signal or object can be uniquely represented in the fingerprint.

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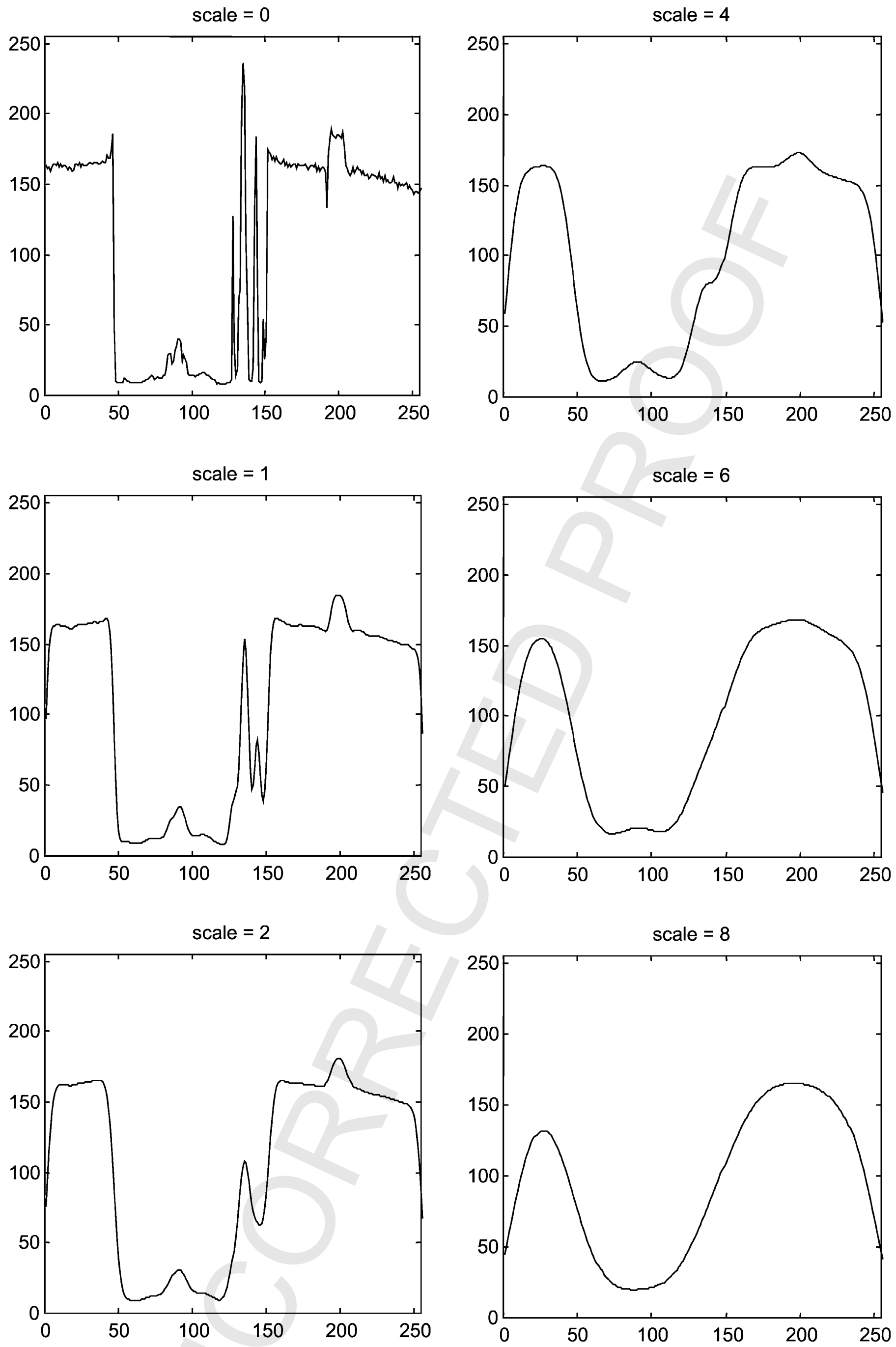

Fig. 1. Selected layers of an 1-D Gaussian scale-space (padded with zeros on either side).

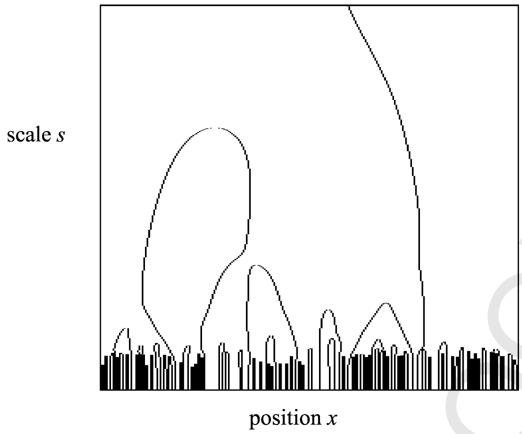

Fig. 2. Fingerprint of the 1-D Gaussian scale-space shown in Fig. 1, with local extrema defined as feature points.

The continuity requirement becomes poorly defined in scale-spaces created from discrete-domain signals and/or when the scale parameter exists only at discrete values. In these cases it may be possible, however, to track a feature from scale to scale and to set a bound on feature localization error.

Finally, it is desired that a scale-space must be generated by an *Euclidean invariant* filtering procedure. Translation and/or rotation of the original image must give an equally translated and/or rotated scale-space. Again, for discrete signals some attempt to approximate this property will need to be implemented, as only a finite set of exact rotations and translations exist on a finitely spaced grid.

Each of these requirements for a set of signal representations to be called a true scale-space originates from the goal of simplification of the original signal. Fidelity and causality require that we begin with the original signal and filter only in the direction of higher scale. The monotone and continuity requirements ensure that the filtering only simplifies the signal, as measured in the set of features. Euclidean invariance then requires that the scale-generating filtering procedure be independent of any prior knowledge about orientation or position of the original signal.

Under these general constraints, many different possibilities exist for scale-space construction. The distinctions arise from the definitions of the scale-generating filters and signal features. As we shall see, the dimensionality of the original signal also has a great effect on possible scale-spaces. Often, the filter and feature definitions that create a scale-space for one-dimensional signals do not create true scale spaces in two or higher dimensions, because one or more of the aforementioned requirements (typically the monotone property) is violated.

3. Linear scale-space and diffusion

Here we review the common scale-generating filters in the image processing literature, beginning with the Gaussian filter. The original Gaussian scale-space theory, proposed by

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Iijima [5] and later by Witkin [6], has been shown to be the only viable *linear* scale-space in image processing [7]. Non-linear scale-space research has generally focused on non-linear partial differential equations (PDEs), such as those used in anisotropic diffusion techniques, and on mathematical morphology. After a brief review of the classical linear scale-space, we concentrate on the morphological scale spaces, which possess particular feature-preserving advantages for image processing applications.

The convolution of a signal with a zero-mean Gaussian kernel creates the Gaussian scale-space. (An example is shown in Fig. 7 at the conclusion of this paper, for the original image of Fig. 6.) The standard deviation of the Gaussian kernel is then the scale parameter. The scale-space generated in this manner obviously satisfies the fidelity requirement, because as the standard deviation goes to zero, the original image is approached. This filtering also satisfies Euclidean invariance, because a spatially symmetric Gaussian is always used. Causality is satisfied, in the sense that each scale is derived solely from the previous scale(s) and not vice versa. In fact, since two convolutions by Gaussian filters in succession is equivalent to one single convolution by a Gaussian (of the summed variance), each scale can be derived directly from the original or directly from any previous scale. The results are identical.

The monotone property is almost always the most difficult scale-space requirement to be met. It has been proven that the Gaussian is the only *linear* convolution kernel that satisfies the scale-space monotone requirement in one dimension (1-D), with zero-crossing of the second derivative defined as features [7,8]. However, in two dimensions (2-D), with zero-crossings of the Laplacian as features, "a closed zero-crossing contour can split into two as the scale increases ..." [8]. This violates the monotone requirement. A similar situation occurs for the 2-D case of local extrema as features, with new extrema being generated at higher scale in certain cases [9].

Finally, the continuity requirement is met in the continuous version of the Gaussian scale-space. However, in the discrete implementation, where the concept of continuity is ill-defined, features must be tracked from scale to scale as they drift in location. (For a more complete discussion of discrete Gaussian scale-spaces, we refer the reader to Lindeberg [10].)

The Gaussian blurring is an example of linear low-pass filtering. Signal "edges," or high gradients, are blurred along with the rest of the signal. This is a drawback with regard to image analysis, as the edges often correspond to the physical boundaries of objects. In most image analysis tasks, it is therefore desired that edges be localized and not drift as scale increases. This drift can be seen as an example of the uncertainty principle, where a low-pass filter (the Gaussian) restricts the frequency spectrum, and thus simultaneously "blurs" the spatial domain (edges) [3]. This is a consequence of linear filtering, and as we will see, there are non-linear scale-generating filters specifically designed to maintain edges through scale.

PDE methods can be also used to generate a Gaussian scale-space. The solutions to the isotropic *diffusion* or "heat" equation [3]

$$\frac{\partial F(\mathbf{x},t)}{\partial t} = \Delta F(\mathbf{x},t), \tag{2}$$

where Δ is the Laplacian, are indeed samples (for fixed time t) of the Gaussian scale space. The time variable t is related to the standard deviation of the Gaussian kernel σ (the scale parameter) by the relation $\sigma^2 = 2t$.

The diffusion equation has been generalized to various non-linear PDEs in order to correct the shortcomings of linear diffusion, such as edge localization. For example, the *anisotropic diffusion* technique of Perona and Malik [11] discourages diffusion in the direction of high gradient magnitude, where presumably important edges occur. They show that the causality and monotone properties are preserved (with properly chosen diffusion coefficients) when features are defined as local extrema.

Many variants of these ideas are possible and non-linear diffusion and PDE scale-space techniques are active areas of research [12–14]. However, these techniques tend to be computationally expensive and often involve a large number of parameters. A more detailed discussion of PDE-based techniques for scale-space generation and edge extraction is given in [15].

In contrast to PDE methods, scale-space techniques based on mathematical morphology tend to be more computationally efficient and require very few arbitrary parameters. Certain morphological filters are especially successful in image processing because of their edge preservation and noise removal properties. For the remainder of this paper, we focus on morphological scale-space techniques. For an analysis of morphological scale-space from a PDE viewpoint, see [16].

4. Morphological scale-space

In this section, we review the fundamental concepts and definitions of mathematical morphology, beginning with sets and binary signals, and then discussing generalization to functions and gray-level signals. Following this review, we present a summary of the prior research on morphological scale-spaces.

4.1. Background on morphology and morphological scale-space

Mathematical morphology is a set-theoretical algebra consisting of two fundamental operators, dilation and erosion. A binary signal can be considered a set A, and erosion and dilation then correspond to Minkowski addition and subtraction by another set B called the *structuring element* [17]. Here we use the notation

$$A \oplus B = \{a + b:\ a \in A,\ b \in B\}, \tag{3}$$

for dilation of a set A by structuring element B. Erosion is then the *dual* operator of dilation

$$A \Theta B = (A^c \oplus B)^c, \tag{4}$$

where A^c denotes the complement of A, given a structuring element B that is symmetric with respect to reflection about the origin.

Further morphological operators are formed as combinations of dilation and erosion. The open operator is defined by

$$A \circ B = (A \Theta B) \oplus B \tag{5}$$

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and its dual, the close operator by

$$A \bullet B = (A \oplus B)\Theta B. \tag{6}$$

Open and close are *idempotent* operations [18]. That is, once an open or close operator is applied to a signal, it is not altered by subsequent application of the same filter.

The above operators are defined for sets or binary signals, but can be generalized to functions or *gray-level* signals. The definitions of the fundamental set operations, dilation and erosion, are generalized to functions by supremum and infimum operations in the equations

$$f(\mathbf{x}) \oplus g(\mathbf{x}) = \sup_{\mathbf{y}\in G}\big\{f(\mathbf{x}-\mathbf{y}) + g(\mathbf{y})\big\} \tag{7}$$

and

$$f(\mathbf{x})\Theta g(\mathbf{x}) = \inf_{\mathbf{y}\in G}\big\{f(\mathbf{x}+\mathbf{y}) - g(\mathbf{y})\big\}, \tag{8}$$

where the structuring element is $g(\mathbf{x}) : G \subseteq R^n \to R$.

Gray-level open and close operators remain as defined above, but in terms of gray-level dilation and erosion [3]. The gray-level (function) operators retain the idempotent property given above for sets. The concatenation of open and close produces other commonly used operators. Open–close and close–open are given by $(f(\mathbf{x}) \circ g(\mathbf{x})) \bullet g(\mathbf{x})$ and $(f(\mathbf{x}) \bullet g(\mathbf{x})) \circ g(\mathbf{x})$, respectively, for a function $f(\mathbf{x})$ and structuring element $g(\mathbf{x})$. Because these filters are serial combinations of open and close filters, they also possess the edge-localization and smoothing properties that make open and close filters useful in image processing. In Section 5, we will see that the open–close and close–open filters have a strong relationship with the local monotonicity of a signal or image.

Morphological scale-spaces are generated using spatially symmetric structuring elements, in accord with the scale-space requirement for Euclidean invariance. Also, we further restrict the structuring element (or structuring function) to be non-positive, to have a maximum value of zero (at the origin), and to be *convex* [3]. In morphology, a convex function is defined by the property that any chord connecting two points lying on the function is contained entirely on or below the function. Unfortunately, in analysis, such a *function* is termed concave [3]. A more mathematically precise statement would then be that the umbra [18] of the structuring function is a metric-convex set [19]. Typical structuring elements meeting these requirements include the constant-valued or “flat disc,” spherical, and parabolic structuring elements. The simplest of these is the flat structuring element having the constant value of zero in a circular region around the origin, and is defined to be $-\infty$ elsewhere. In this simple case, it should be noted that gray-level morphological operations are equivalent to their binary counterparts acting on each of the *level-sets* generated by binary thresholds applied to the signals.

By filtering a gray-level signal with scaled structuring elements, scale-spaces can be formed. Scale-spaces may be generated by a number of different combinations of morphological operators. In the literature, previous studies have focused on those created by dilation or erosion [3], close or open [20,21], and close–open or open–close [22]. The dual of any operator that generates a scale-space generates another scale-space, or

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a dual scale-space. A morphological scale-space generally differs from its dual scale-space by its bias in gray-level direction. For example, dilation produces signals that are everywhere brighter than the original, while erosion has the opposite effect. Often in specific applications, one scale-space method is chosen for use over the corresponding dual. This decision may be based on knowledge about the particular application (e.g., tracking dark objects in a bright image), but is often arbitrary. We wish to develop a scale-space that does not depend on a priori knowledge of the signal. That is, we wish to avoid the *gray-level bias* introduced by the choice of one filter over its dual.

Some attempts to equally merge a scale-space with its dual have been proposed [3]. As we will see, the scale-spaces created from close–open or open–close filtering can be viewed as an attempt to minimize the disparity between a scale-space and its dual scale-space, thus minimizing the gray-level bias. After a review of these previous studies, a morphological scale-space without gray-level bias is introduced in Section 5.

4.2. Dilation–erosion scale-space

Jackway [3] proposes a basic morphological scale-space, one derived by application of a scaled dilation or erosion operator. This scale-space is referred to as the dilation–erosion scale-space, and examples are shown in Figs. 8 and 9. In contrast to other scale-spaces, the scale parameter here is allowed to be negative. Positive scales are generated by application of scaled dilation operators to the original signal, and the erosion operator generates "negative scales."

The introduction of this non-standard range of scale parameter possesses some advantages, but violates most of the standard scale-space requirements. By considering the scale-spaces created by dilation and erosion together, the scale-space is not gray-level biased towards a certain intensity direction. The scale-space generated by the original signal $+f(\mathbf{x})$ is equal to the negative of the scale-space generated by $-f(\mathbf{x})$. This lack of gray-level bias is desirable for a general scale-space technique, where no prior knowledge of the signal is assumed. However, since both positive and negative "scales" are simplifications derived from the original signal, our previous scale-space requirements (such as causality) require some refinements in order for dilation–erosion to qualify as a true scale-space.

The dilation–erosion scale-space meets the scale-space requirements of Euclidean invariance and fidelity (under our previous assumptions regarding allowed structuring elements). For all digital signals, and for bounded continuous signals, it can be shown that $F(\mathbf{x}, s) \to f(\mathbf{x})$ as $s \to 0$ [3]. That is, the original signal corresponds to zero scale. Here, the introduction of a negative scale parameter causes no difficulty. The proof for the dilation operator $(s > 0)$ extends to the erosion operator $(s < 0)$, so that $F(\mathbf{x}, s)$ approaches the original function from both directions.

Before considering the other scale-space requirements, it is necessary to define the features of dilation–erosion scale-space. Because dilation and erosion correspond to local supremum and infimum operations, local extrema are the most natural candidates for features in this scale-space.

The monotone property of features is notoriously the most difficult scale-space requirement to satisfy. Here, the features are signal extrema, and a modified monotone property can be shown. Local maxima are considered features for positive scale, and local minima

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are the features for negative scale. If P_s is the set of all features at scale s, then for positive scales

$$\forall a > b > 0, \quad P_a \subseteq P_b. \tag{9}$$

Similarly, for negative scales

$$\forall a < b < 0, \quad P_a \subseteq P_b. \tag{10}$$

By considering the absolute value $|s|$ as the true scale parameter, this monotone property can be more simply written as $|a| > |b| > 0$, $P_a \subseteq P_b$. The monotone property expressed here actually satisfies continuity and *strong* causality, as well. Features that exist at higher scale $|s|$ not only exist at all lower $|s|$ (including the original signal), but exist at the same spatial location. Note, however, that for this monotone property to hold, it is necessary to further restrict the choice of structuring element such that it has a single-point local maximum. For example, a flat structuring element would cause a point-sized feature (local maximum) in the original signal to become a many-point plateau of features at higher $|s|$.

It should be noted, though, that simply taking absolute values of the scale parameter does not completely reconcile dilation–erosion with the scale-space monotone requirement. Because features are defined differently depending on the sign of s, dilation–erosion is more akin to two different scale-spaces. Thus, allowing the scale parameter to assume negative values may simply be a matter of convenient notation when displaying a scale-space representation or fingerprint, and not a fundamentally new concept in scale-space theory. Neither dilation nor erosion is an invertible operation, and both filters remove information and simplify the signal. Both remove more features as larger structuring elements are used. Therefore, for all practical purposes, only positive scale—in the true sense of scale—can exist.

Despite the somewhat awkward combination of two scale spaces by the use of positive and negative scale parameters, dilation–erosion scale-space possesses some very desirable properties. Simple feature extraction and strong causality make implementation very efficient, as features can be located and tracked through scale trivially. However, the ultimate test of this scale-space will be in its application to real problems. By defining local extrema as features, this technique is obviously highly sensitive to noise [3]. A single impulse can dramatically alter the scale-space representation of a signal at high scale (after repeated dilations). This sensitivity of higher scales to small perturbations of the original signal is counterintuitive to the concept of scale-space, and remains a serious issue in the use of dilation–erosion as a scale-space.

Another implementation issue concerns the dependence of the discrete dilation–erosion scale-space on the scale of the original signal. Each scale of the dilation–erosion scale-space is formed by dilation (erosion) of the original image with a scaled structuring element. Here, we mention that the scale-space could alternatively be obtained *recursively*, with each scale derived by dilation (erosion) of the previous layer by a small or "differential" structuring element. In fact, in the continuous case, these scale-spaces would be identical.

As mentioned, rotationally invariant structuring elements are used to satisfy the scale-space requirement of Euclidean invariance. As an example, consider the case of flat circular structuring elements. The equality between the dilation scale-space derived directly from

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the original image $f(x)$ and that derived recursively can be shown by the associative property of dilation [18],

$$f(\mathbf{x}) \oplus g_{a+b}(\mathbf{x}) = f(\mathbf{x}) \oplus \big(g_a(\mathbf{x}) \oplus g_b(\mathbf{x})\big) = \big(f(\mathbf{x}) \oplus g_a(\mathbf{x})\big) \oplus g_b(\mathbf{x}). \tag{11}$$

Here, g_a is a circular structuring element of radius a. Note that this equality relies upon the associative property of dilation [18], and upon the semi-group property of circular structuring elements $g_{a+b}(\mathbf{x}) = g_a(\mathbf{x}) \oplus g_b(\mathbf{x})$. Viz., recursive dilation by small circular structuring elements is equivalent to a single dilation by a larger circular structuring element.

However, in discrete implementation there is a difficulty with the recursive approach, because rotationally symmetric structuring elements such as the circular disc cannot be exactly represented on a square grid. The smaller the radius, the more distorted the approximation. For dilation–erosion scale-space, this means that recursive generation of each scale from its immediate predecessor amplifies this error.

Therefore, repeated dilation or erosion by small structuring elements is not equivalent to a single dilation by a structuring element of a larger circular approximation. Recursively generated dilation–erosion scale-space is a poor approximation to the continuous case. This difficulty of discrete approximation may conflict with the scale-space notion of causality. Though each scale depends only on previous scales, the explicit dependence of the entire scale-space upon knowledge of the original scale seems to undermine the spirit of scale-space theory. Ideally, as in the partial differential equation (PDE) methods, each scale should depend only on the previous scale or set of scales.

4.3. Open and close scale-spaces

Concatenation of dilation and erosion lead to the open filter and the dual, the close filter. Unlike a single dilation or erosion, the open and close filters have the desirable property of edge localization, meaning that simple step edges or ramp edges tend to be preserved by the open or close filter. While edges tend to be preserved, impulsive noise and small extrema tend to be removed. The open filter removes positive-going impulses, while the close filter removes negative-going impulses [23]. Thus, the open and close filters simplify a signal representation by filtering out small detail while retaining important edges. This is the essential goal of scale-space theory for image processing. It is important to note, however, that open and close do possess gray-level bias, as did dilate and erode, and that open and close also create two distinct scale-spaces (see Figs. 10 and 11, for example).

An open or close scale-space can be created in a similar fashion to the erosion or dilation scale-space, with each scale is formed by filtering the original signal. It should be noted, however, that unlike the erosion-dilation scale-space, this scale-space can equivalently be generated recursively from one scale to the next, even in discrete implementation. This equivalence is due to the following property of the open filter [18]:

$$\big(f(\mathbf{x}) \circ g_a(\mathbf{x})\big) \circ g_b(\mathbf{x}) = f(\mathbf{x}) \circ g_b(\mathbf{x}), \quad \forall b \geqslant a \geqslant 0, \tag{12}$$

where $g_a(\mathbf{x})$ is a structuring element of scale a meeting our previous requirements. A similar equation can be written for the close operation.

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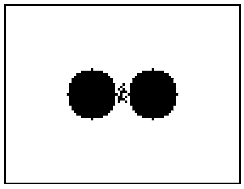

Fig. 3. A simple 2-D example of a saddlepoint situation.

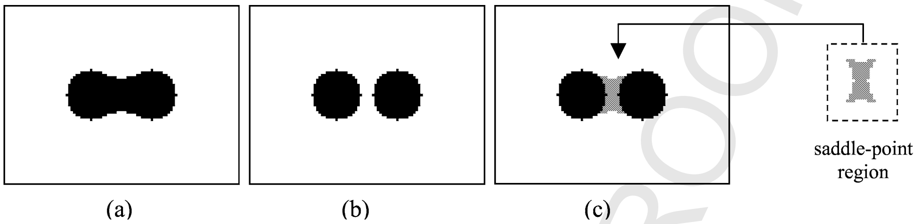

Fig. 4. Fig. 3 after (a) the open (or close–open) filter of structuring element equal in size to the circular object ($r = 10$ pixels), (b) the close (or open–close) filter, (c) the lomo filter with the same structuring element.

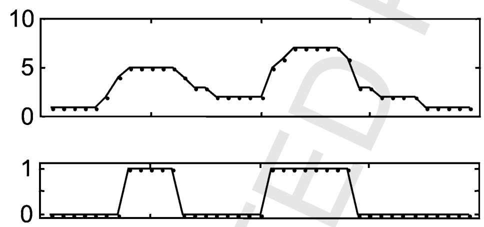

Fig. 5. A discrete 1-D lomo-6 signal $f(x)$ and the level-set (or binary threshold) for $f(x) \geqslant 5$.

For the open filter scale-space, Park and Lee [20] define feature points as zero-crossings in the second derivative for 1-D signals. They show that a monotone property does exist, provided that some caution is used at the so-called *angular points*, where the second derivative does not exist. This refinement of the feature point definition becomes trivial for discrete signals, where an approximation for the second derivative is utilized.

It is natural to attempt a generalization of the 1-D feature definition to 2-D open and close scale-spaces, by defining features to be the zero-crossings of the Laplacian. Unfortunately, the monotone property no longer holds [20]. A simple example for the close filter scale-space is shown for the artificial images of Figs. 3 and 4. This image shows an example of a recurring problem in 2-D scale-space theory, a saddle-point region, which we consider in further detail in Section 5. For the present illustration of features in the close filter scale-space, consider Fig. 4a to be the original image. Here, the zero-crossings of the Laplacian happen to lie on the boundaries between the constant-valued object and the constant background. Applying the close filter with a flat structuring element of increasing scale eventually forms the coarser scale representation shown in Fig. 4b. Again, the features lie on the boundaries between dark and light regions. Thus, the zero-crossing features are closed contours that can divide in two as the scale increases. As mentioned previously,

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Fig. 6. Original bell tower image, 256 × 256 pixels and 256 gray-levels.

Fig. 7. Selected scales from the Gaussian scale-space generated from Fig. 6. From top, standard deviations of 1, 2, and 4 pixels.

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Fig. 8. Dilate scale-space derived from Fig. 6 using structuring elements of radius 1, 2, and 4 pixels (from top).

Fig. 9. Dilate scale-space derived from Fig. 6 using structuring elements of radius 1, 2, and 4 pixels (from top).

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Fig. 10. Close scale-space derived from Fig. 6 using structuring elements of radius 1, 2, and 4 pixels (from top).

Fig. 11. Open scale-space derived from Fig. 6 using structuring elements of radius 1, 2, and 4 pixels (from top).

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this division of feature contours also occurs in the 2-D Gaussian scale-space and seems to violate the scale-space monotone requirement.

Chen and Yan [21] propose another 2-D generalization, but for continuous-domain *binary* images. For the boundary of each binary *connected-component* within the image, a 1-D function can be formed representing the radius of curvature vs. arc length. By reducing the representation to 1-D, features can then be defined as zero-crossings of this curvature function. At angular points in the curvature function, Nacken [24] shows that zero-crossings can be introduced with increasing scale, thus violating the scale-space monotone requirement. To restore the monotone property, a discrete approximation to zero-crossings must be used [20,25] to overcome the under-counting of features. Chen and Yan also describe a discrete implementation for measuring the boundary curvature. With this correction, the number of zero-crossings of this curvature is shown to be non-increasing as scale increases, so the scale-space monotone requirement is met. Park and Lee have attempted the generalization of this technique from binary to gray-level images [20]. However, they do not provide feature definitions that satisfy the monotone requirement.

To reduce gray-level bias, though not to eliminate it, the concatenation of open and close can be used to construct scale-spaces. Applying the open–close or close–open filters to the original image can generate open–close and close–open scale-spaces, or each scale can be generated from the previous scale. As mentioned above, continuous-domain dilation–erosion and open/close scale-spaces can be equivalently derived in either manner. However, this is not the case for open–close and close–open scale-spaces. In order to reduce sensitivity to noise, and reduce gray-level bias, the recursive scale-space generation is greatly preferred. That is, each scale is generated from the previous scale by the application of open–close (or close–open) filters of increasing radius. This recursive scale generation is referred to as *alternating sequential filtering*. Examples of scale-spaces generated in this manner are shown in Figs. 12 and 13.

4.4. Connected operators and area morphology

In 1-D, the open–close or close–open filters, using flat symmetric structuring elements, are equivalent to another class of non-linear filters called *sieve* filters [22], though this equivalence breaks down in higher dimension. Sieve filters can be extended to higher dimensions by interpreting them as connected operators, which act on the connected-components of the level-sets (see, for example, [26,27]). One such sieve filter is the area sieve, by which connected-components below a given area threshold are removed (or volume threshold in 3-D, etc.). This filtering process is often referred to as *area morphology*, with area open and area close filters removing level-set connected-components of “1’s” and “0’s,” respectively. In 1-D, area simply refers to segment length, and flat structuring elements are also specified by this segment length, so the open and close filters of standard morphology in the previous section are equivalent to the connected operators of area morphology.

In two dimensions, however, the rigid structuring elements of standard morphology are replaced by completely deformable structuring elements specified only by area and not by shape. This difference can be described as a difference between “solid” and “liquid” structuring elements, the former imposing its own structure upon the filtered connected-

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Fig. 12. Close–open alternating sequential scale-space derived from Fig. 6 (structuring elements of radius 1, 2, and 4 pixels).

Fig. 13. Open–close alternating sequential scale-space derived from Fig. 6 (structuring elements of radius 1, 2, and 4 pixels).

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components and the later conforming to the shapes already present in the signal by way of geodesic operators [28,29]. Thus, scale-spaces formed by replacement of standard open and close filters by their area morphology (or area sieve) counterparts are more faithful to the original signal and tend to retain boundaries and features better through scale. This property is advantageous in applications such as segmentation [27].

However, along with less restrictive filtering and greater fidelity to the original signal come less precise scale properties. For example, because a connected-component object may take on any shape so long as it contains the required area, regular sampling of a signal after area morphological filtering may not accurately capture the object in a reconstruction. Accurate reconstruction from sampling is not necessarily a scale-space requirement, but we assert that sampling and signal scale should be related. Also, because two regions of particular connected-component may be joined by a single pixel, it is possible for area morphological scale-spaces to be highly noise-dependent and unstable in the sense that small variations in the signal may produce dramatically different scale-spaces. Some experimental results show that the filters are in fact robust in the presence of noise [13]. However, this potentially global dependence upon single pixels, even if only in rare cases, goes against our intuitive notions of local formulation of scale-spaces. Nonetheless, for many applications, the feature-retaining benefits of area morphology may outweigh the potential shortcomings of these pathological cases. Examples of alternating sequential scale-spaces using area open–close and area close open are shown in Figs. 14 and 15.

One popular compromise between the behavior of standard morphology verses that of area morphology is morphology by reconstruction [30]. While standard morphology employs a solid structuring element and area morphology a liquid structuring element, morphology by reconstruction uses a "melting" structuring element. First, connected-components are removed if they cannot enclose a solid structuring element, just as in standard morphology. However, unlike standard morphology, the surviving connected-components are left unaltered. Rather than forcing the solid structuring elements to everywhere fit within the shape of the connected-component, the structuring element "liquefies" and conforms to any connected-component shape. Opening and closing by reconstruction act on "1" and "0" components, respectively.

Bangham et al. [22] show that discrete-domain 1-D sieve filtering satisfies the scale-space monotone requirement with edges as features. In this case, edges include any point not equal to the corresponding neighbor to the right. The open–close and close–open filters replace local extrema with flat segments. Thus, edges are only removed and never created. However, the application of 1-D sieve filters in multiple directions (e.g., horizontal and then vertical) does not ensure an equivalent monotone property. In this case, edges (differences between neighboring points) may occur in multiple directions. The removal of an edge in one direction by an open or close filter may create an edge in a different direction at that same point. 2-D sieves with rigid structuring elements also fail to preserve the scale-space monotone requirement.

However, 2-D area sieves are able to preserve the scale-space monotone requirement for both extrema and edges as features [31]. For the monotone requirement to hold for gray-level functions, edges are defined simply as differences in gray-level intensity between adjacent pixels. As scale increases, these gray-level differences do not increase in magnitude and never change in sign. The removal of connected-components below

Fig. 14. Area close–open alternating sequential scale-space derived from Fig. 6 (area of 5, 13, and 49 pixels).

Fig. 15. Area open–close alternating sequential scale-space derived from Fig. 6 (area of 5, 13, and 49 pixels).

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the area threshold simply leaves flat zones, and thus removes both extrema and edges. In particular, the set of edge pixels satisfies strong causality be retaining precise edge localization. For the case of extrema as features, these flat zones, or plateaus, may then combine or divide in two as scale further increases using standard morphology (e.g., see Figs. 3 and 4, similar to the example described in [9] for the Gaussian scale-space). Again, however, area morphology does not suffer from this violation of the monotone property, as flat zones cannot divide with increasing scale.

The ability of connected operators to maintain scale-space monotone property can be exploited in additional ways. For example, Jackway is able to combine aspects of dilation–erosion scale-space, connected operators, and gradient watersheds in a scale-space that preserves a monotone property [25]. First, we recall that watershed regions can be extracted from a gray-level image by interpreting the gradient magnitude as a topographical surface [32,33]. Basins then correspond to smooth image regions of low gradient, and watershed boundaries represent edges or high gradient magnitudes. Jackway uses connected operators on this gradient magnitude function to remove basins that are not marked. The connected operators remove unmarked level-set connected-components but cannot create new watershed regions due to the monotone property of the connected operators. However, rather than using an area measure as a marker, intensity extrema in the original image serve as markers. This way, the monotone property of intensity extrema that exists for dilation–erosion scale-space (Section 4.2) ensures such a property for the watershed regions themselves.

Thus, by generalizing concepts of standard morphology to allow for connected operators without rigid structuring elements, the elusive scale-space monotone requirement can be retained for higher-dimensional signals, not only for the case of area morphology, but for even more general connected operators.

5. The lomo scale-space

While the area morphological operators of the previous section can be utilized in scale-space generation which meet the desirable requirements outlined in Section 2, we introduce a scale-space that addresses some additional scale properties that have not been explicitly stated, but which we believe are implied in the notion of scale. These include the concept of local formulation, whereupon the scale-generating operator acts upon a spatially localized window rather than arbitrarily extended regions of connected-components. Additionally, we support the notion of stability, by which single values cannot significantly affect the scale-space, as is possible in the case of connected operators. And finally, we desire a scale-space formulation that exhibits no gray-level bias and avoids the need for dual scale-spaces. It is this bias that we address first.

Each of the morphological scale-space filters reviewed in Section 4 exhibits a gray-level bias, which leads to a bias toward either darker or brighter intensities in the scale-space. Dilation and the close filter are *extensive*, meaning that the filtered signal is everywhere greater than or equal to the original. Extensive filters are therefore biased towards higher

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intensity. Erosion and the open filter are *anti-extensive*, and similarly biased towards lower intensity. For a gray-level signal $f(\mathbf{x})$, these relationships can be summarized by

$$f(\mathbf{x}) \Theta g(\mathbf{x}) \leqslant f(\mathbf{x}) \circ g(\mathbf{x}) \leqslant f(\mathbf{x}) \leqslant f(\mathbf{x}) \bullet g(\mathbf{x}) \leqslant f(\mathbf{x}) \oplus g(\mathbf{x}). \tag{13}$$

The gray-level bias of the open and close filters is easily observed in the presence of impulsive noise. The open filter removes (smoothes) positive-going impulses, while the close filter removes negative-going impulses. Similarly, signal extrema are treated unequally by open and close filtering.

The open–close and close–open filters are can be viewed as attempts to reduce the gray-level bias of the open and close filters, by combining an extensive and an anti-extensive operation. Nonetheless, open–close and close–open filters are not equivalent and still exhibit gray-level biases.

Here, we wish to propose a non-linear scale-generating filter with no gray-level bias. One filter without gray-level bias is the median filter, which simply outputs the median value of a windowed set of samples [34]. Median filters have noise removal and edge localization properties that are similar to those of morphological filters [35,36]. However, the median filter has two major disadvantages. First, when repeatedly applied to a signal (to obtain a median root signal), the median filter is susceptible to oscillations and generates streaking and blotching artifacts [37]. Second, convergence to a root signal in the multi-dimensional case is not guaranteed.

In addition to the noise filtering and edge localization of median filtering, it is known that median filter root signals are *locally monotonic* in 1-D. Local monotonicity is a desirable smoothness property with a well-defined scale parameter associated with it. The 1-D definition of local monotonicity and our proposed 2-D generalization are discussed in detail in the next section. The concept of local monotonicity is the motivation for our introduction of the morphological lomo filter.

Morphological approximations to the median filter have been proposed, including filters that are not gray-level biased, i.e., that are *self-dual* [38]. Among the self-dual filters are the linear combinations of dual morphological filters. The filter created by averaging the outputs of the dilation and erosion filter is the mid-range filter [39]. The filter created by averaging the outputs of open and close filters has been referred to as the pseudo-median filter [40,41]. Similarly, the average of open–close and close–open filters has been proposed [42].

We introduce an iterative scale-generating filtering procedure based on the linear combination of the dual morphological filters open and close. The self-dual filtering procedure is designed to create locally monotonic root signals at multiple scales. In order to define this scale-space, we begin with a discussion of local monotonicity in the 1-D and multi-dimensional cases.

5.1. Local monotonicity in 1-D

A 1-D *locally monotonic* signal of degree n, or *lomo-n* signal, is defined as a signal that is monotonic (either non-increasing or non-decreasing) in every subinterval of a given length n [36]. In higher dimensions, there is no generally accepted definition. A locally monotonic signal is unaffected by an open or close filter whose structuring element is

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flat, spatially symmetric, and of length n for continuous signals (length $n-1$ for discrete signals) [18,23]. Thus, a locally monotonic signal is a root signal of both open and close filters (and thus open–close and close–open as well).

The degree (or window size) of local monotonicity n depends on the size of the structuring element [18,23]. For scale-space generation, the scale parameter can be defined by the radius r of the symmetric structuring element rather than its length n, allowing a more convenient generalization to higher dimensions. Let the structuring element be centered at a given point and cover all points in the signal with distance from the center is less than or equal to r. For continuous-domain signals this implies that $r = n/2$, and for discrete signals that $r = (n-2)/2$. For discrete signals, we will only consider integer scales and thus n must be an even integer.

In addition to the relationship between local monotonicity and 1-D open/close morphological filters, several scale-related properties of lomo-n signals can be established. First, there exist plateaus between ascending and descending intervals. Between any increasing interval and any decreasing interval, there must exist a constant interval of length $\geqslant n$ (continuous case) or $\geqslant n-1$ (discrete case) [36]. Any local minima or maxima is a member of adjacent ascending and descending intervals, and therefore must be contained within a plateau. Figure 5 shows a lomo-6 signal.

Also, level-set connected-components have a minimum size. 1-D lomo signals are roots of the open and close filters of the structuring element described above, and each connected-component in each level-set of is at least large enough to enclose this structuring element. (Proof: Level-set objects must contain at least one local extremum, and the plateau property of extrema sets a lower bound on the size of the connected-components within a level set.) Because the filtering is self-dual, this statement applies equally to both 1 and 0 level-set components. This means that there is a minimum size or *lomo scale* associated with all connected-component objects identified in the level-sets of a lomo-n signal. This property is illustrated in Fig. 5.

These 1-D properties follow directly from the original definition based on non-increasing and non-decreasing segments. However, in higher dimensions, non-increasing and non-decreasing become ill-defined concepts. Should the 2-D signal be non-increasing along *any* path of length n, any *straight* path, or along at least *one* straight path? We provide a multi-dimensional generalization of the 1-D local monotonicity concept that attempts to retain the important 1-D properties, while providing a specific filtering procedure for generating locally monotonic signals.

In 1-D, there is a filtering procedure for the generation of locally monotonic signals of a specified degree or scale. Though iterative median filtering is susceptible to oscillation, a morphological method is possible. Due to the fact that the open and close filters are idempotent, the open–close always produces a root signal of the close filter, while the close–open produces a root signal of the open filter. The open–close and close–open filters themselves are also idempotent [17] in 1-D. In fact, in 1-D, both open–close and close–open filters produce root signals of *both* open and close in a single pass [23], and are thus locally monotonic.

In 2-D, however, open–close (or close–open) filtering does *not* always produce a root signal of both open and close simultaneously. The example in Figs. 3 and 4 demonstrates this shortcoming. Thus, the precise specification of signal smoothness (local monotonicity)

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that was possible for 1-D open–close scale-space is not possible in 2-D. This discrepancy is one of the main motivating factors behind the proposed lomo filter, along with the gray-level bias present in open–close and close–open filters.

5.2. The lomo filter and local monotonicity in multiple dimensions

Let a *lomo filter* be defined by the update equation

$$f(\mathbf{x}) \leftarrow \frac{f(\mathbf{x}) \circ k(\mathbf{x}) + f(\mathbf{x}) \bullet k(\mathbf{x})}{2}, \tag{14}$$

for a function $f(\mathbf{x})$ and *flat* symmetric structuring element $k(\mathbf{x})$ (which meets the same requirements as for the other morphological scale-spaces). It should be noted that a single application of the lomo filter is *not* idempotent, and (14) is written as an iterative update equation. We propose to continue iteration at each scale until a root signal is reached, then likewise at the next coarser scale, similar to an alternating sequential filtering. By proceeding recursively from one scale to the next, a scale-space is generated.

By utilizing both open and close filters, this filtering method is robust in the presence of impulsive noise, as is alternating sequential filtering. We purposely choose the fixed structuring elements of standard morphology rather than the less restrictive area in order to impose more specific scale properties in multiple dimensions, such as local formulation and stability as discussed in the previous section. Additionally, the use of fixed structuring elements allows for morphological sampling as described by Haralick [43], and therefore multi-resolution processing for those applications where efficient storage and computation are necessary. We assert that these properties help the proposed method to adhere more faithfully to the traditional concepts of scale-space than do area morphological or other connected operator formulations, and that the retention of these properties outweigh the distortion imposed by the fixed shape of the structuring element. The scale properties of multi-dimensional lomo signals are to follow in the remainder of this section. An example of this lomo scale-space is shown in Fig. 16.

The iterative filtering procedure is used to generate a root signal of the lomo filter at each level. In 1-D, this root signal is a root of both open and close filters and is thus locally monotonic. The generalization of the lomo filter to multiple dimensions facilitates a definition of local monotonicity for images and videos.

We propose that a multi-dimensional signal be defined as *strict-sense lomo-n* if and only if it is a root signal of the multi-dimensional open and close filters simultaneously. This definition is consistent with the 1-D definition and the relationship between n and the structuring element radius r is identical to (**??**) and (**??**).

The specification of "strict-sense" in the above definition allows room for a less restrictive generalization based on the lomo filter. In 1-D, the lomo filter is observed to converge on a strict-sense lomo signal. However, in 2-D the filter fails to converge to a root of both open and close filters at certain signal locations. In 2-D, a function can possess *saddle-points* surrounded by regions where the lomo-filter converges to a root signal that is neither a root of the open filter nor of the close filter. An example is given in Figs. 3 and 4. Consider Fig. 3 to be the original image, and Fig. 4 to be the corresponding lomo root image, for a structuring element equal in size to each dark object in Fig. 4.

Fig. 16. Lomo scale-space derived from Fig. 6 using (14) with structuring elements of radius 1, 2, and 4 pixels (from top).

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This treatment of saddle-point regions avoids the gray-level bias of other morphological scale-spaces. However, as with the other morphological scale-spaces, if zero-crossings of the Laplacian are defined as scale-space features, the closed contours may divide as scale increases. Thus, the 2-D scale-space monotone property remains an unresolved problem for the lomo scale-space as well.

We propose then to use the general term *lomo-n* to refer to any signal which is a root signal of the lomo filter, though not necessarily of the open and close filters everywhere. This causes no conflict with the 1-D definition, as in 1-D saddle points cannot occur, and it retains the useful properties of local monotonicity when generalized to higher dimensions. This less restrictive definition of 2-D local monotonicity is preferable for actual applications, because it provides a filtering procedure for converting a non-lomo input signal to a lomo signal.

Many important 1-D properties of locally monotonic signals are retained in this generalization to higher dimensions. First, any local extremum is contained within a constant region such that the structuring element fits within that region. This gives these constant regions a minimum size (radius). A minimum size of all level-set connected-components is also retained in 2-D. Here, saddle-point regions must be considered where the signal is not a root of open and close filters. However, there can be no level-set object contained entirely within such a saddle-point region, because such regions contain no local extrema, whereas all level-set objects must contain at least one extremum. Thus, all level-set points in saddle-point regions are connected to plateau regions that satisfy the minimum size requirement. These are the well-defined scale properties that make lomo scale-space generation possible.

In 1-D, the roots of the lomo filter are median filter roots and a morphological approximation to median filtering has been the goal of research on self-dual filters including the linear combination of open and close [38,41,42]. However, in 2-D, root signals of the lomo filter are not necessarily median roots, and as previously stated they are not necessarily open and close roots either. Nonetheless, the lomo filter possesses the desirable noise-removing and edge localization properties that are the motivation for both the median filter and the open and close morphological filters.

While the iterate given by (14) satisfies our need for a filter that generates lomo signals, it is noteworthy that this filter is not necessary unique. For example, a root signal of (14) is also a root signal of both

$$f \leftarrow \frac{(f \circ k) \bullet k + (f \bullet k) \circ k}{2} \tag{15}$$

and

$$f \leftarrow \frac{((f \circ k) \bullet k) \circ k + ((f \bullet k) \circ k) \bullet k}{2}. \tag{16}$$

(In (15) and (16), we omit the argument $\mathbf{x}$ in the notation for $f(\mathbf{x})$ and $k(\mathbf{x})$ for convenience.) Here we present a brief outline of the proof. Assume f is a root signal of (14):

$$f \circ k + f \bullet k = 2f. \tag{17}$$

Then,

$$f \bullet k = f + (f - f \circ k) = f + \rho_\circ, \tag{18}$$

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where $\rho_\circ = f - f \circ k$ is the open residue. Applying open to (18) gives

$$(f \bullet k) \circ k = (f + \rho_\circ) \circ k. \tag{19}$$

Since the addition of the open residue of f to a signal f does not affect subsequent open filtering, we obtain

$$(f + \rho_\circ) \circ k = f \circ k. \tag{20}$$

Combining (19) and (20), and demonstrating the dual case for the close operation ($(f \circ k) \bullet k = (f - \rho_\bullet) \bullet k = f \bullet k$), the filter in (15) reduces to the filter in (14). The same property extends this proof to include the filter in (16).

So, each of these lomo filters may serve as an equivalent *definition* of local monotonicity of a given scale, i.e., that they share the same set of root signals. However, it should be noted that iterative application of each of these filters leads to different lomo roots within that root set. Visually, there are only very subtle differences. It is our desire here to merely note that these filters may provide equivalent definitions of local monotonicity, while providing slightly different lomo filters for creating lomo signals from non-lomo signals. Alternative lomo filters may be advantageous. For example, (14) attenuates an image impulse at a geometric convergence rate, while (15) eliminates the impulse in a single iteration.

A natural application of the lomo scale-space is image segmentation. Like scale-space filtering, segmentation is a low-level process that is important to many higher-level applications in computer vision. The generation of a lomo scale-space has many advantages as a pre-processing step in image segmentation. The lomo filter serves as a useful smoothing filter, with well-defined signal-smoothing properties that retain edge localization and remove of small extrema. This should allow for scale selection and edge detection, two important needs of image segmentation. In current research, we are developing a segmentation technique that is optimally matched with the lomo scale-space [44].

6. Comparison of scale-spaces

The lomo scale-space compares favorably with the other above-mentioned scale-spaces in terms of its fidelity to the original image. This is shown experimentally in Table 1, where the mean squared error (MSE) of various scale-spaces and scales is computed with respect to the original image of Fig. 6. Dilate and erode scale-spaces exhibited significantly greater MSE than the alternating sequential scale-spaces due to gray-level bias and the drifting of edges. Surprisingly, the open and close scale-spaces appear to outperform these less biased scale-spaces. However, this is simply due to the lack of filtering in one gray-level direction. For example, the open filter removes small-scale positive-going regions while leaving small-scale negative-going regions unaffected. Thus, at a given scale, the open filter may only alter a signal approximately half as much as the open–close filter. The superior MSE of the open and close scale-spaces is at the expense of poor removal of small-scale features.

Also included in the comparison are area morphological scale-spaces corresponding to their alternating sequential fixed structuring element counterparts. Here, the only equivalent scale parameter between the two cases is the area of the structuring element.

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Table 1
Mean squared error (from the original 256-gray-level image of Fig. 6) of the various scale-spaces and scales shown in Figs. 8–16. For area morphological filters, area corresponds to that of the fixed structuring element (i.e., 5, 13, and 49 pixels)

Scale-space filtering	Structuring element size (scale)		
method	Radius = 1 pixel	Radius = 2 pixels	Radius = 4 pixels
Dilate	165.0	427.0	1039.2
Erode	169.8	443.0	1037.9
Close	17.8	43.0	136.8
Open	19.8	67.0	192.6
Close–open	29.3	63.9	153.4
Open–close	29.5	76.9	191.3
Area close–open	4.0	8.5	25.0
Area open–close	4.0	8.5	25.0
Lomo	27.1	59.8	113.7

Table 2
Mean squared error (from the original 256-gray-level image of Fig. 6) of scale-space images derived from the corruption of Fig. 6 by additive white Gaussian noise, $\sigma = 10$ gray-levels. (Images not shown.) For area morphological filters, area corresponds to that of the fixed structuring element (i.e., 5, 13, and 49 pixels)

Scale-space filtering	Structuring element size (scale)		
method	Radius = 1 pixel	Radius = 2 pixels	Radius = 4 pixels
Dilate	317.1	686.8	1490.8
Erode	344.0	749.8	1606.7
Close	85.9	159.0	385.5
Open	93.3	191.0	455.6
Close–open	66.9	86.9	130.0
Open–close	66.9	99.6	159.4
Area close–open	34.6	29.7	41.7
Area open–close	35.2	30.3	42.8
Lomo	41.4	67.3	116.1

Therefore, for comparison, the area of the structuring element is retained, but the shape is allowed to deform in the area morphological case. Therefore, the area morphological scale-spaces are significantly closer to the original image for a given area. However, as can be seen in the figures, the sense of scale is barely perceptible compared to the fixed structuring element methods. Thus, it is difficult to compare directly area morphological methods to fixed structuring element methods by attempting to relate the scale parameters.

When noise is present, the alternating-sequential and recursive scale-spaces (close–open, open–close, and lomo) consistently outperform the strictly extensive and anti-extensive scale-spaces (dilate, erode, open, and close) in terms of MSE. Table 2 shows the MSE of the scale-spaces derived from a noise-corrupted image. Here, zero-mean white Gaussian noise with a standard deviation of 10 gray-levels is added to the original 256-gray-level image of Fig. 6. By removing both positive and negative-going noise, the alternating sequential and lomo scale-spaces easily outperform the others in terms MSE. The lomo scale-space, with no gray-level bias, is the most faithful to the original

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image of the fixed structuring element methods in terms of MSE at each scale. The area morphological scale-spaces again are the closest to the original, or least filtered. For this image, we observe no instability of these area morphological scale-spaces due to noise, and for an area parameter of 13 pixels noise is removed effectively as reflected in the MSE values.

While MSE should certainly not be taken as the sole criterion in the preference of one scale-space method over another, it does represent a valid measure of one important aspect of scale-space generation, the fidelity to the original image. The lomo scale-space, with its fidelity to the original image through scale, lack of gray-level bias in removing small-scale features, robustness in the presence of noise, and well-defined scale properties should be considered an attractive alternative to previous fixed structuring element morphological scale-spaces.

7. Conclusions

The state of the art in morphological scale-space has been reviewed, and a scale-space has been proposed that builds upon past scale-space research. The proposed method attempts to retain desirable aspects of previous work in 1-D morphological scale-spaces, especially local monotonicity and edge localization, when generalized to higher dimensions. At the same time, the new method tries to overcome many of the difficulties that exist in prior methods, particularly the problem of gray-level bias. The scale-space will enable efficacious solutions to image segmentation, object-based image coding and coarse-to-fine searches.

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Joseph H. Bosworth received the B.S. degree in physics from the Massachusetts Institute of Technology in 1992 and the M.S. degree in physics from Yale University in 1993. He is currently pursuing a doctorate degree in electrical engineering at the University of Virginia, where he is a research assistant. His research interests include multiscale image segmentation, scale-space theory, and mathematical morphology.

Scott T. Acton received the B.S. degree from the Virginia Polytechnic Institute and State University in 1988, and the M.S. and Ph.D. degrees from the University of Texas at Austin in 1990 and 1993, all in electrical engineering. He has worked in industry for AT&T, the MITRE Corporation, Motorola, Inc. Currently, Dr. Acton is an Associate Professor in the Department of Electrical Engineering at the University of Virginia. He is an active participant in the IEEE, ASEE, SPIE and Eta Kappa Nu and an associate editor of the *IEEE Transactions on Image Processing*. His research interests include multiscale image representations, diffusion algorithms, image morphology, and image restoration.